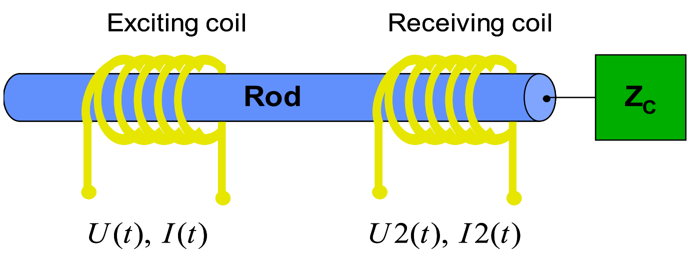
Case studies and Optimization



■ Initialization

■ Values for case studies and optimization

■ General values

All values are given in standard SI units!

Permeability of Vacuum (w/o units):

```
val0 = {
  μ0 -> VacuumPermeability / (Volt Second / (Ampere Meter))}
```

$\left\{\mu 0 \rightarrow \frac{\pi}{2500000}\right\}$

Properties of the rod (radius of coil = radius of solenoid).

```
val1o = {
  ER → 207 * 10^9,          (*E-modulus of rod in Pa*)
  cR → 4824,                (*sound velocity in meter/second*)
  λ → -15 * 10^6,           (*Magnetostrictive constant in xxxxxxxxxx*)
  μrR → 80}                 (*relative permeability rod*)
```

```
{ER → 207000000000, cR → 4824, λ → -15000000, μrR → 80}
```

Properties of the function generator:

```
val2o = {
   U0d → 10,            (*Amplitude in Volt (NOT peak-peak!)*)
   Ri → 50}             (*Internal resistance in Ohm*);
```

```
val3o = {
    A → π r^2,
    μR → μ0 * μrR} /. Flatten[{val0, val1o}];
```

```
valueso = Flatten[{val1o, val2o, val3o}]
```

$$\{ER \to 207000000000,\ cR \to 4824,\ \lambda \to -15000000,\ \mu rR \to 80,\ U0d \to 10,\ Ri \to 50,\ A \to \pi r^2,\ \mu R \to \frac{\pi}{31250}\}$$

Now the values are applied onto the coefficients a1 and a2, generating new rules:

```
valuerulea1o = rulea1 /. valueso;
```

```
valuerulea2o = rulea2 /. valueso;
```

```
generalruleao = Flatten[{valuerulea1o, valuerulea2o}]
```

$$\left\{a1 \to -\frac{201\, Ud\, \left(-207000000000\, \pi\, r^2 + 207000000000\, \pi\, r^2 \operatorname{Cos}\left[\frac{lRod\, w}{4824}\right] + 4824\, i\, ZP \operatorname{Sin}\left[\frac{lRod\, w}{4824}\right]\right)}{575\, N\, \pi\, r^2\, w^2 \left(4824\, ZP \operatorname{Cos}\left[\frac{lRod\, w}{4824}\right] + 207000000000\, i\, \pi\, r^2 \operatorname{Sin}\left[\frac{lRod\, w}{4824}\right]\right)},\ a2 \to \frac{201\, i\, Ud}{575\, N\, \pi\, r^2\, w^2}\right\}$$

- **HP FGen**
- **Example Rods:**
- **Example contact impedance**

A cured epoxy resin together with a sealant could have an impedance of:

```
ZCexample1 = 5 + 5 + 10^-11 f^2 - 900000 i / f;
```

Partially cured (E-modulus of 0.5 GPa):

```
ZCexample2 = 5 + 3 + 10^-11 f^2 - 400000 i / f;
```

■ Case studies

No real optimization is performed to determine values for the following variables. However, the following case studies show tendencies.

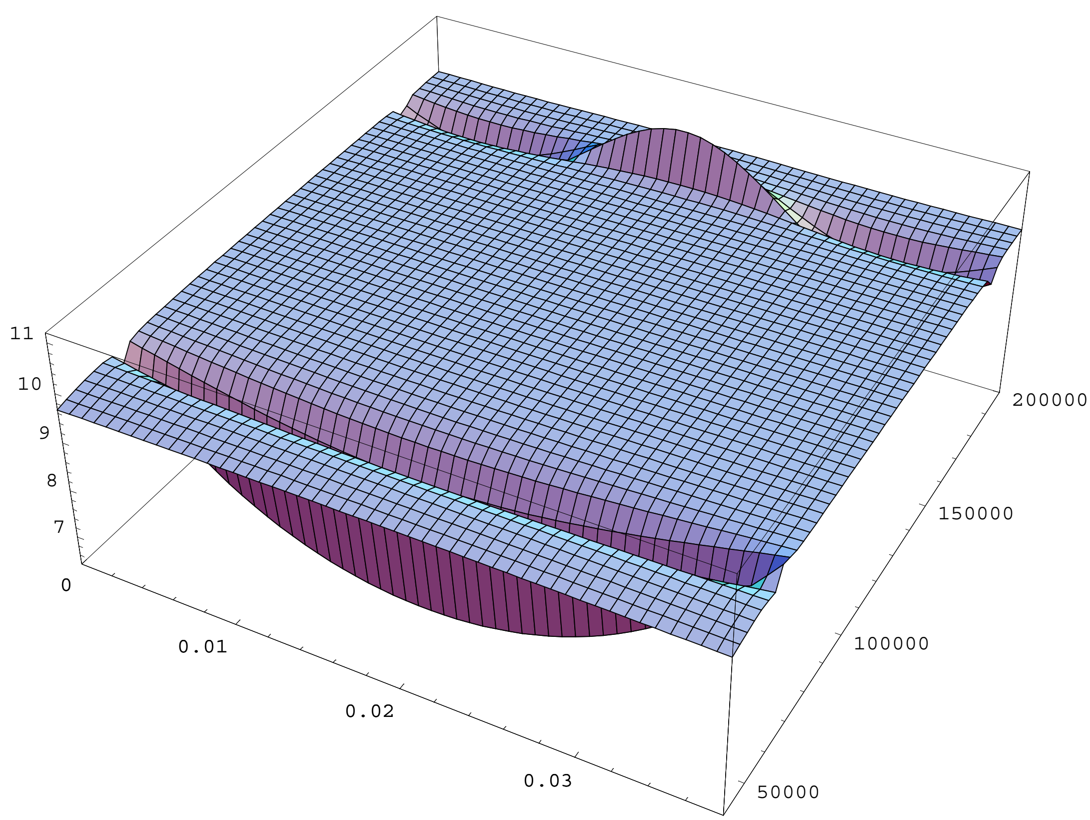
■ Excitation coil position

```
case3general = Flatten[{lCoil1 → 1 / 20 * lRod, examplerod1, generala,
    generalruleUd, valueso, w → 2 π f,  ZC → ZCexample1, N → 80, K1 → 2 / 3}];
```

■ Voltage

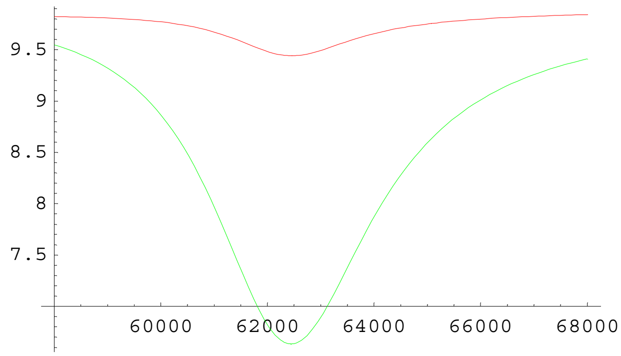
```
testU = Abs[Ud //. case3general] // N;
```

```
Plot3D[testU, {x1, 0, lRod - lCoil1 //. case3general},
 {f, 40000, 200000}, PlotPoints → 50, PlotRange → All]
```




```
- SurfaceGraphics -
```

```
Plot[{testU /. x1 → 0.00, testU /. x1 → 1 / 3 * lRod - lCoil1 / 2 //. case3general},
 {f, 58000, 68000},
 PlotRange → All, PlotPoints → 100,
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```

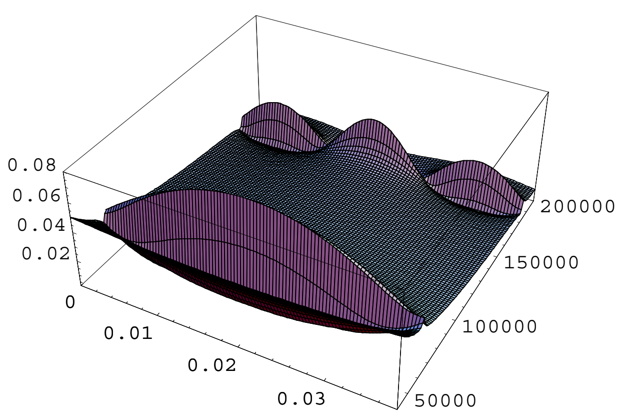


- Graphics -

■ Current

```
testI = Abs[ia[t] / T[t] //. case3general // N];
```

```
Plot3D[testI, {x1, 0, lRod - lCoil1 //. case3general},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → All]
```

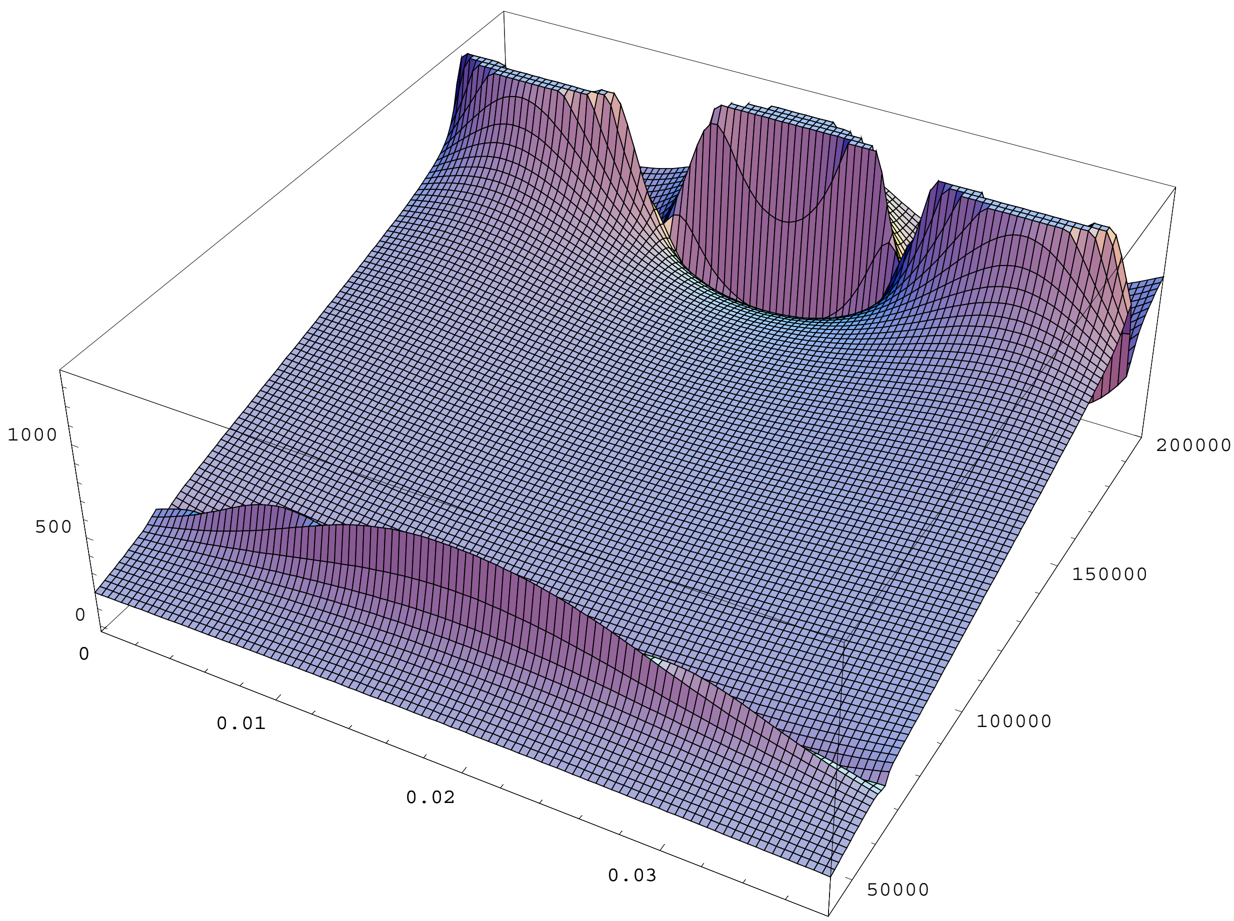


- SurfaceGraphics -

■ Impedance

```
testZ = Abs[Z] //. case3general // N;
```

```
Plot3D[testZ, {x1, 0, (lRod - lCoil1) //. case3general},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → Automatic]
```



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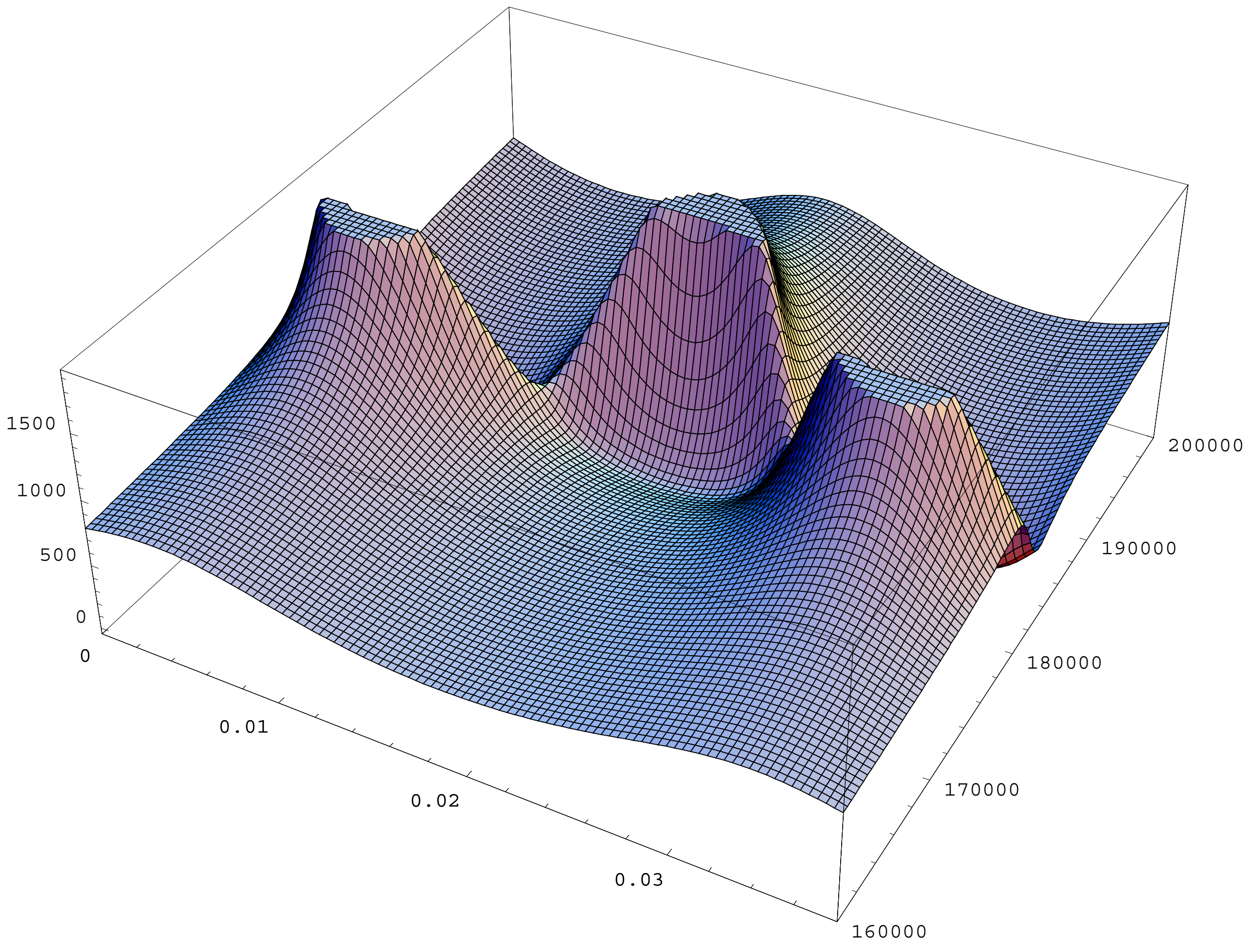
```
Plot3D[testZ, {x1, 0, (lRod - lCoil1) //. case3general},
 {f, 160000, 200000}, PlotPoints → 100, PlotRange → Automatic]
```

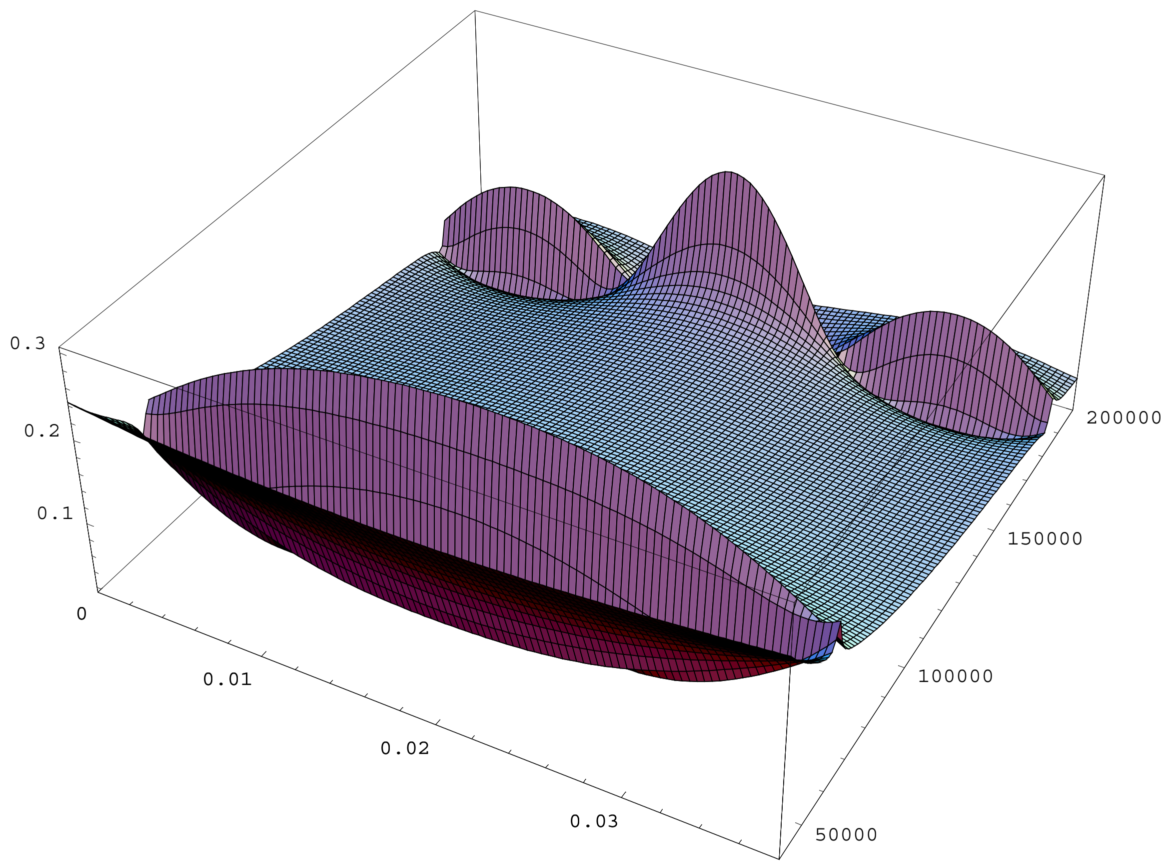



```
- SurfaceGraphics -
```

- **The apparent magnetostrictive power transferend to the rod:**

```
testP = Pa //. case3general // N;
```

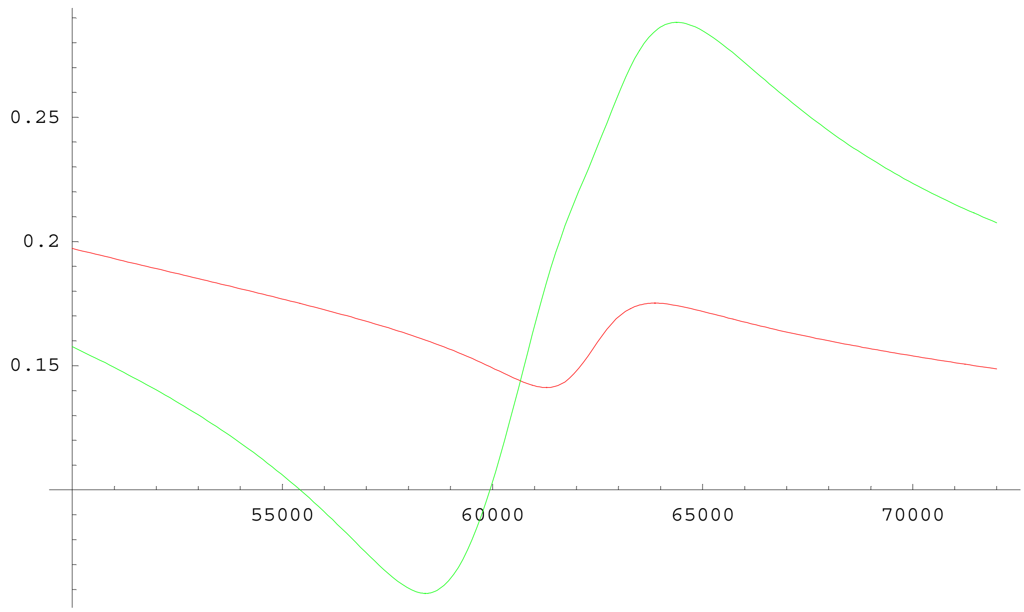
```
Plot3D[testP, {x1, 0, lRod - lCoil1 //. case3general},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → All]
```




```
- SurfaceGraphics -
```

Now for exact placement of coil at 0 and 1/3 (two cuts from the former graph):

```
Plot[{testP /. x1 → 0.00, testP /. x1 → 1 / 3 * lRod - lCoil1 / 2 //. case3general},
 {f, 50000, 72000},
 PlotRange → All, PlotPoints → 200,
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```



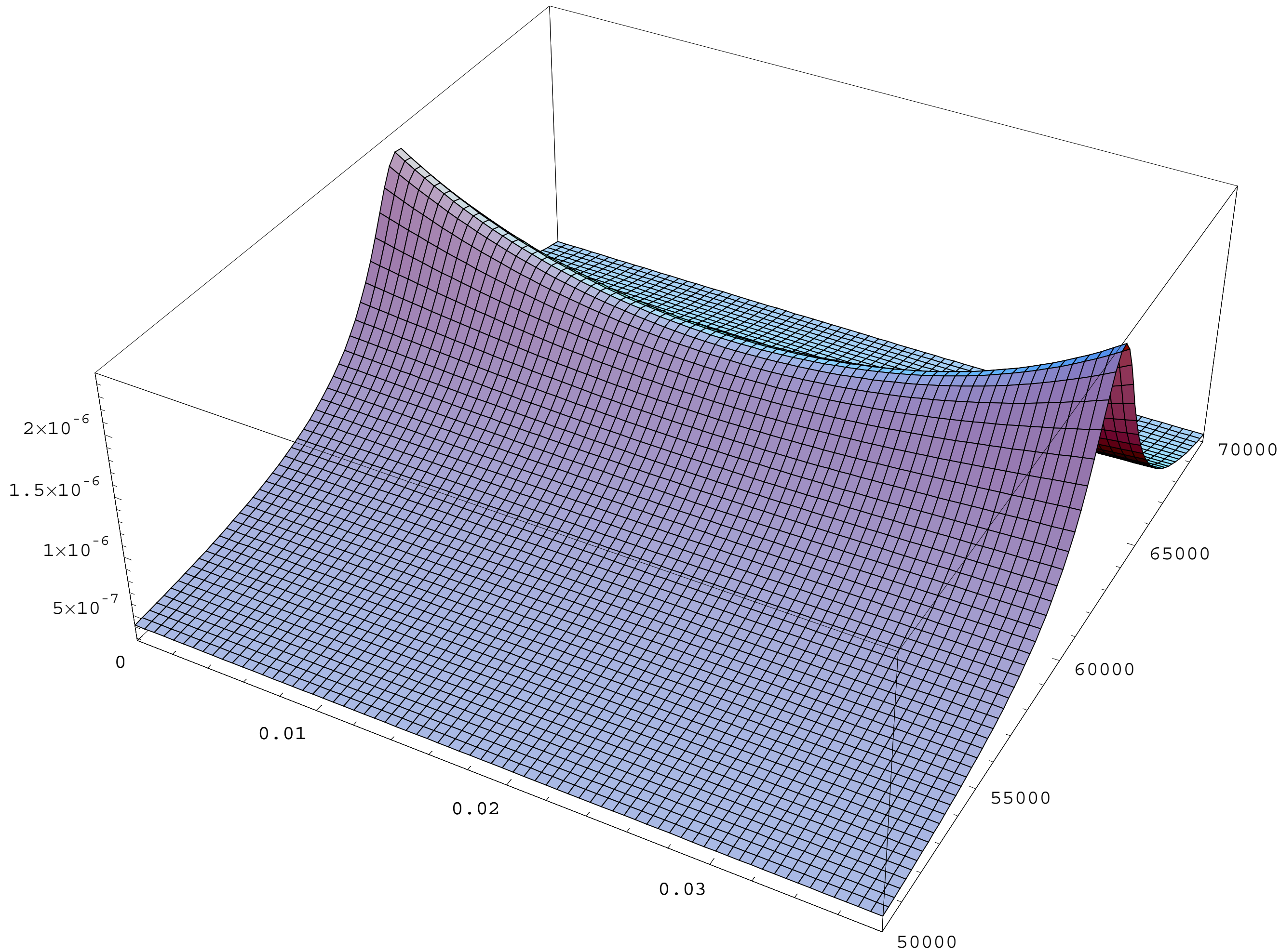

```
- Graphics -
```

■ Amplitude variation dependant on coil position

For the following case the absolute amplitude for a short coil (much shorter than the wavelength in the rod) is calculated. The two variables for the surface plot are coil position and frequency. The contact impedance is zero.

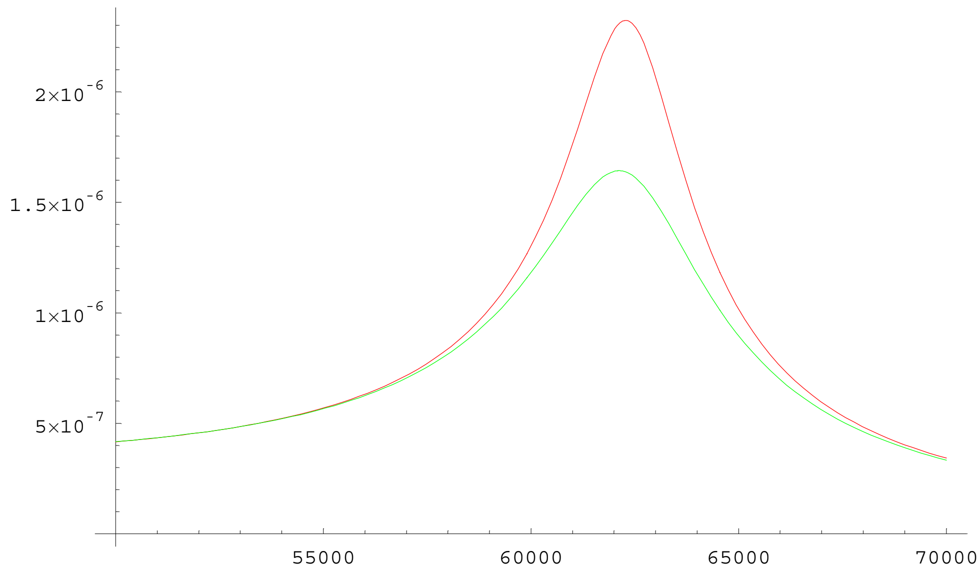
```
testA = ComplexExpand[
    ComplexExpand[amp[a1, a2] //. case3general, x1, TargetFunctions -> {Re, Im}]] // N;
```

```
Plot3D[testA, {x1, 0, lRod - lCoil1 //. case3general},
 {f, 50000, 70000}, PlotPoints → 70, PlotRange → All]
```




```
- SurfaceGraphics -
```

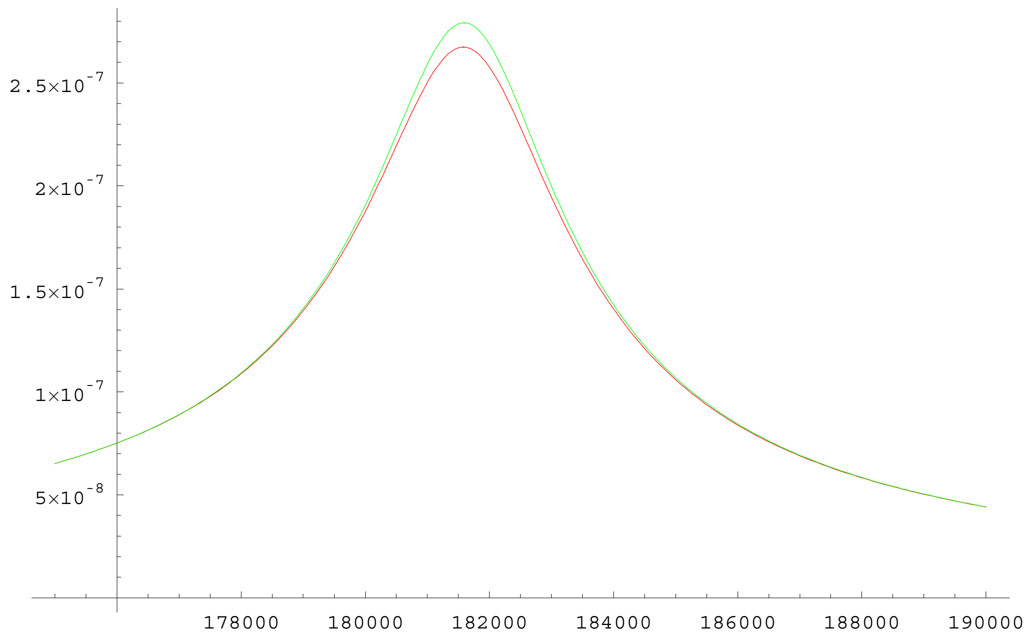
```
Plot[{testA /. x1 → 0.00, testA /. x1 → 1 / 3 * lRod - lCoil1 / 2 //. case3general},
 {f, 50000, 70000},
 PlotRange → Automatic, PlotPoints → 100,
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```



- Graphics -

Now at 3rd mode:

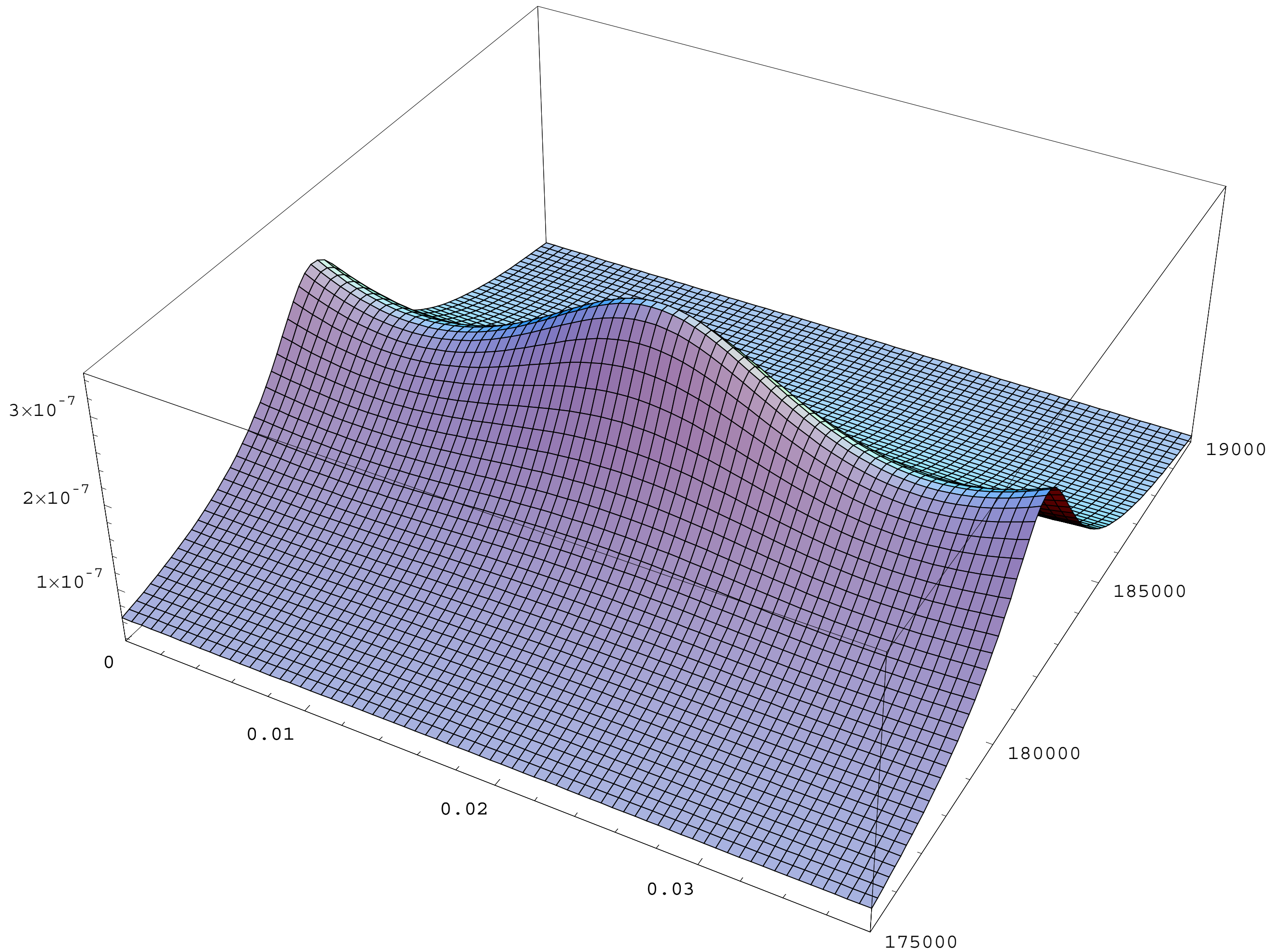
```
Plot[{testA /. x1 → 0.00, testA /. x1 → 1 / 3 * lRod - lCoil1 / 2 //. case3general},
 {f, 175000, 190000},
 PlotRange → Automatic, PlotPoints → 200,
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```




```
- Graphics -
```

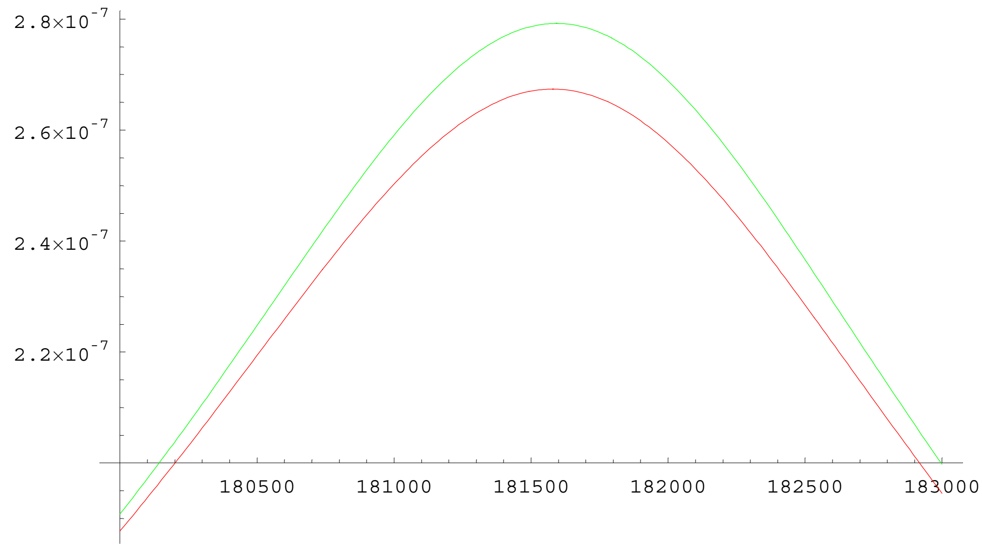
Amplitude @ 3rd mode:

```
Plot3D[testA, {x1, 0, lRod - lCoil1 //. case3general},
 {f, 175000, 190000}, PlotPoints → 70, PlotRange → All]
```



- SurfaceGraphics -

```
Plot[{testA /. x1 → 0.00, testA /. x1 → 1 / 3 * lRod - lCoil1 / 2 //. case3general},
 {f, 180000, 183000},
 PlotRange → Automatic, PlotPoints → 200,
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```




```
- Graphics -
```

■ Receiving coil signal dependant on excitation coil position

■ Which coil should be at the free end?

The drawing shows the exciting coil at the free end of the rod. However, the following case compares this to another configuration, where the coils changed their positions. Both coils have identical length, but a different number of turns.
Obviously the order of the coils makes only a difference if the contact impedance Zc≠0. Therefore Zc is set to a fictive value.

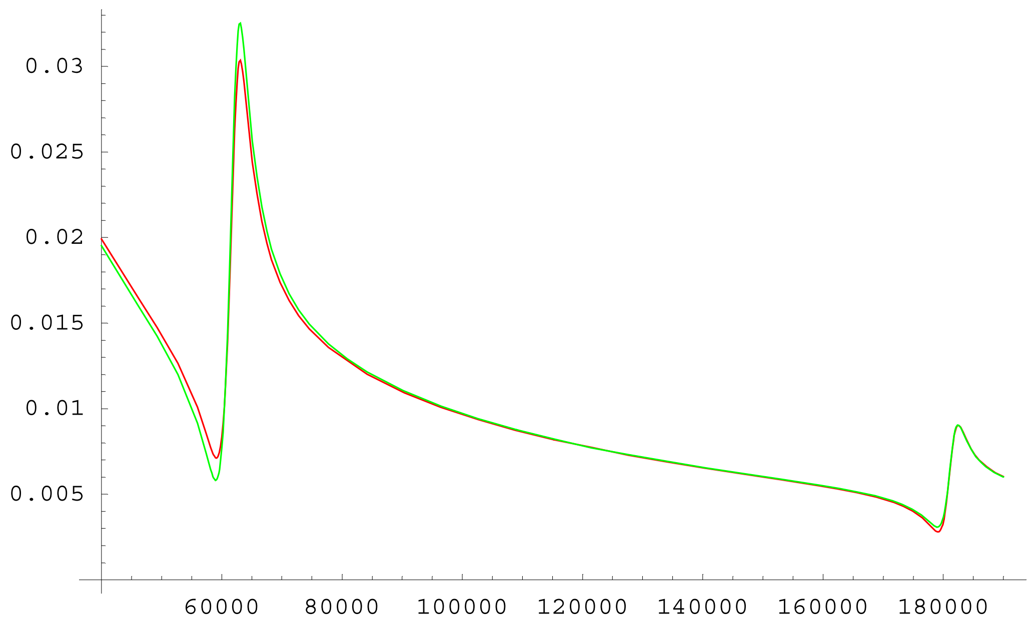
```
case2general = Flatten[
   {lCoil1 → 1 / 2 * lRod - 6 / 1000, lCoil2 → lCoil1, N → 500, N2 → 150, r → 1 / 1000, generala,
    generalruleUd, valueso, w → 2 π f, K1 → 2 / 3, K2 → K1, lRod → 40 / 1000, ZC → ZCexample1}]
```

The Power of the receiving coil is:

```
testP2 = P2a //. case2general // N;
```

Now two cases. Case 1 (red) is with the excitation coil at the free end:

```
Plot[{testP2 /. x2 -> 1 / 2 * lRod + x1 /. x1 → 3 / 1000 //. case2general,
  testP2 /. x1 -> 1 / 2 * lRod + x2 /. x2 → 3 / 1000 //. case2general},
 {f, 40000, 190000}, PlotStyle → {{Thickness[.002], RGBColor[1, 0, 0]},
   {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.004], RGBColor[0, 0, 1]},
   {Thickness[.004], RGBColor[1, 0, 1]}, {Thickness[.004], RGBColor[0.2, 0.8, 0]},
   {Thickness[.004], RGBColor[0, 1, 0]}, {Thickness[.004], RGBColor[0, 0.6, 0.4]}},
 TextStyle → {FontSize → 12}, GridLines → {None, None}]
```



- Graphics -

P2case11=Abs[P2[t]/.ruleUd/.x1→1/8*lRod/.x2→5/8*lRod//.case2general/.N→200/.f→64000]//N

0.208046

P2case12=Abs[P2[t]/.ruleUd/.x2→1/8*lRod/.x1→5/8*lRod//.case2general/.N→200/.f→64000]//N

0.210302

The difference in the power output is:

Simplify[P2case11-P2case12//N,f>0]

-0.00225543

This shows a excitation coil at the free end is in this case a better combination, which causes a 17mW (3%) higher output.

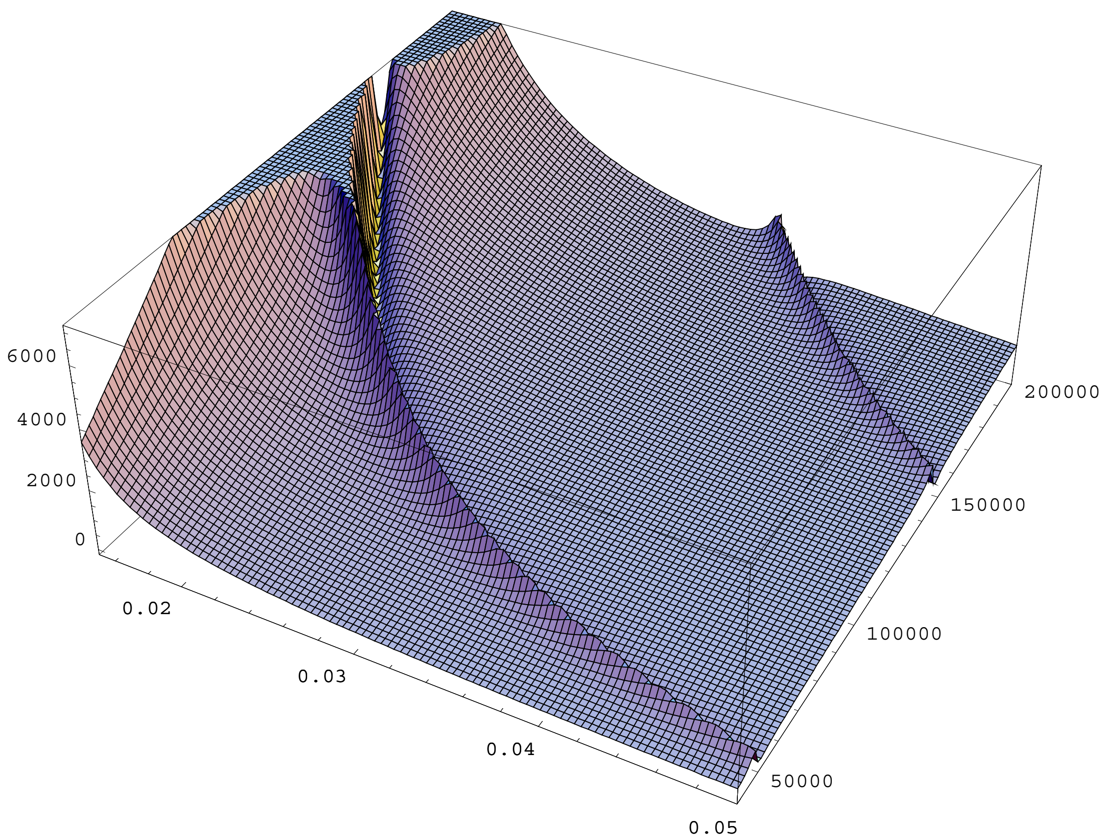
■ Receiving coil length and number of turns

Rod length

```
case5general = Flatten[{lCoil1 → 1 / 2 * lRod - 6 / 1000, lCoil2 → lCoil1,
    x1 → 3 / 1000, x2 -> 1 / 2 * lRod + x1, N → 300, N2 → N, r → 1 / 1000, generala,
    generalruleUd, valueso, w → 2 π f,  ZC → ZCexample1, K1 → 2 / 3, K2 → K1}];
```

```
testZ = Abs[Z] //. case5general // N;
```

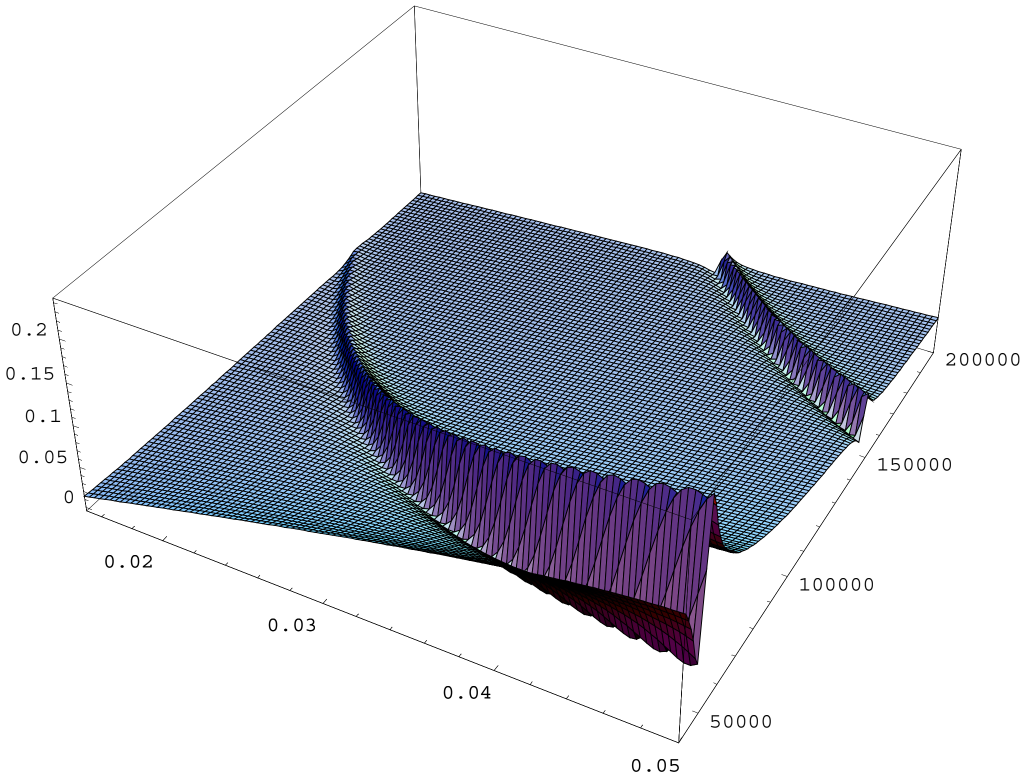
```
Plot3D[testZ, {lRod, 15 / 1000, 50 / 1000},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → Automatic]
```




```
- SurfaceGraphics -
```

```
testP = Pa //. case5general // N;
```

```
Plot3D[testP, {lRod, 15 / 1000, 50 / 1000},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → All]
```




```
- SurfaceGraphics -
```

This shows: the longer the better.

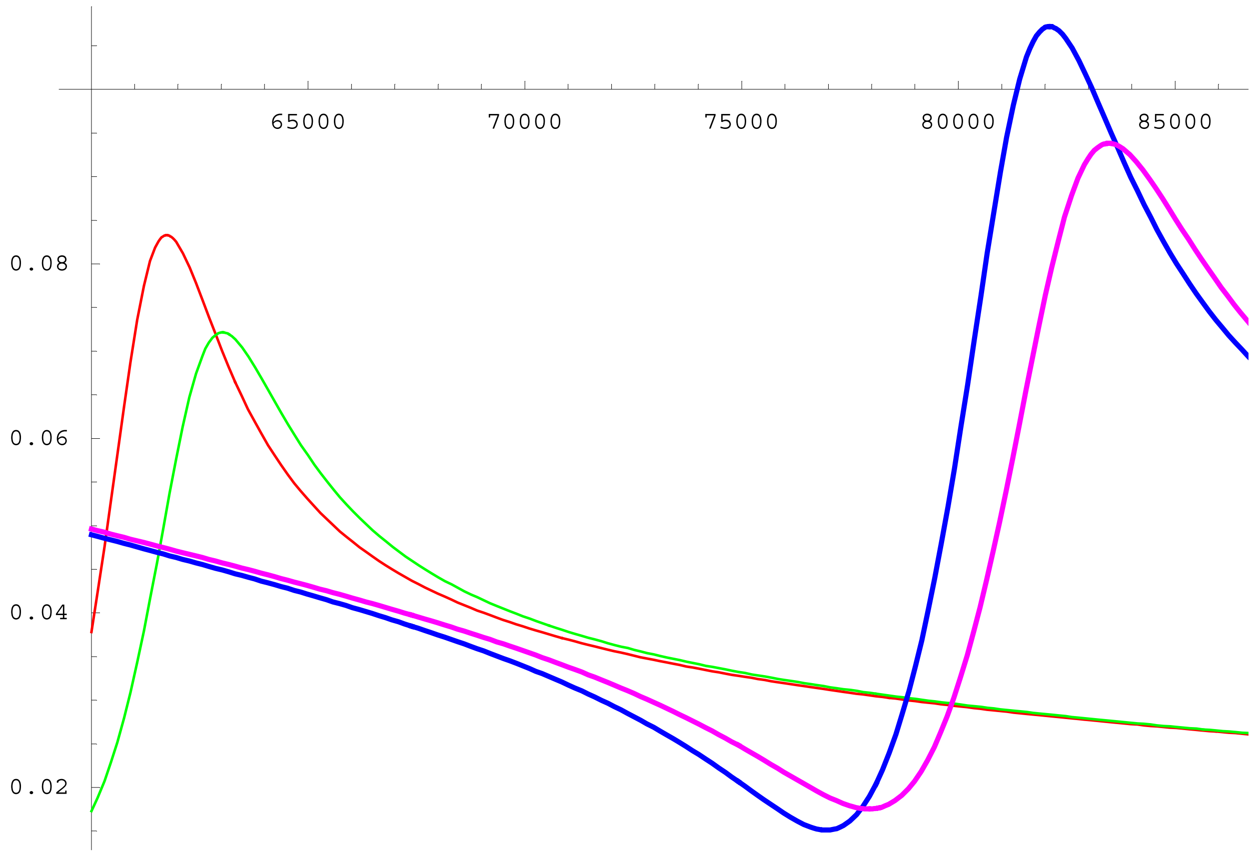
Now finding the biggest frquency-shift:

```
case7general =
  Flatten[{lCoil1 → 1 / 2 * lRod - 6 / 1000, lCoil2 → lCoil1, x1 → 3 / 1000, x2 -> 1 / 2 * lRod + x1,
    N2 → N, r → 1 / 1000, generala, generalruleUd, valueso, w → 2 π f, K1 → 2 / 3, K2 → K1}];
```

```
testPc = Pa //. case7general /. lRod → 40 / 1000 /. N → 500 // N;
```

```
testPd = Pa //. case7general /. lRod → 30 / 1000 /. N → 300 // N;
```

```
Plot[{testPc /. ZC → ZCexample2, testPc /. ZC → ZCexample1, testPd /. ZC → ZCexample2,
  testPd /. ZC → ZCexample1}, {f, 60000, 90000}, PlotRange → All, PlotPoints → 200,
 PlotStyle → {{Thickness[.002], RGBColor[1, 0, 0]},
   {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.004], RGBColor[0, 0, 1]},
   {Thickness[.004], RGBColor[1, 0, 1]}, {Thickness[.004], RGBColor[0.2, 0.8, 0]},
   {Thickness[.004], RGBColor[0, 1, 0]}, {Thickness[.004], RGBColor[0, 0.6, 0.4]}},
 TextStyle → {FontSize → 12}, GridLines → {None, None}]
```




```
- Graphics -
```

long rod: dF= Hz =%

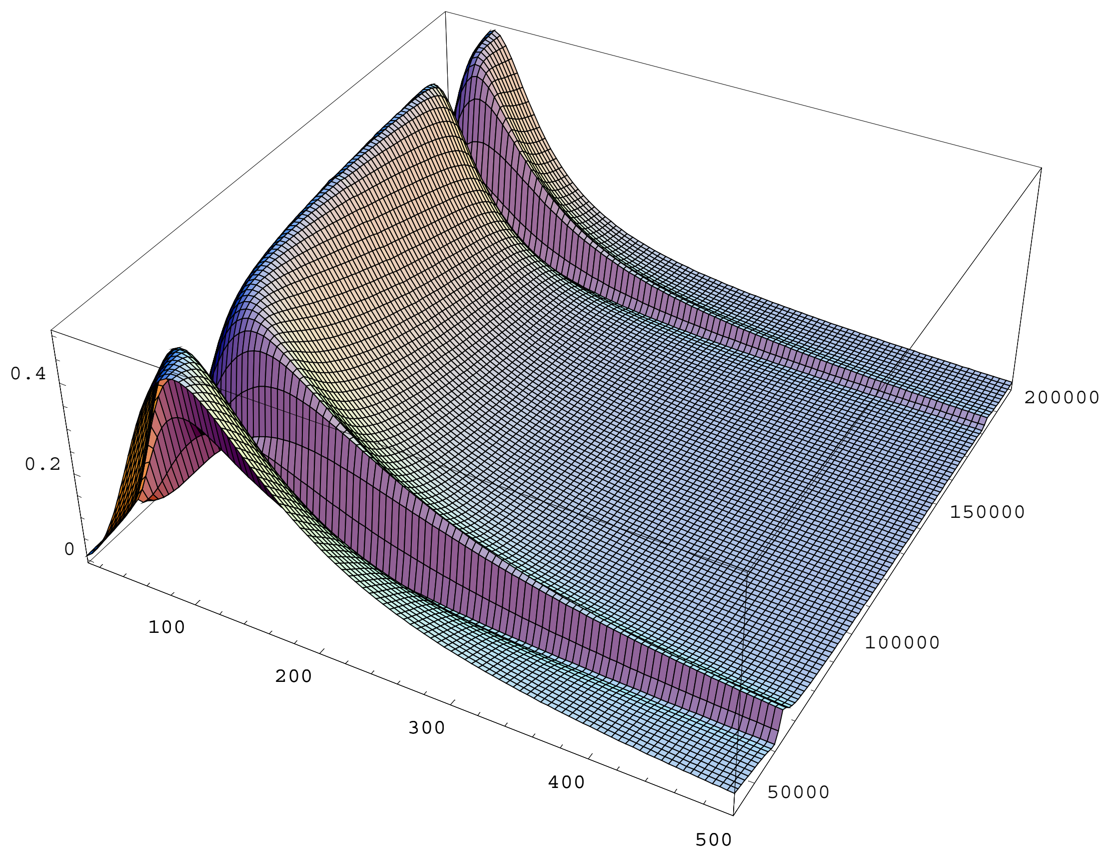
```
- Graphics -
```

```
case6general = Flatten[{lRod → 40 / 1000, lCoil1 → 1 / 2 * lRod - 6 / 1000,
    lCoil2 → lCoil1, x1 → 3 / 1000, x2 -> 1 / 2 * lRod + x1, N2 → N, r → 1 / 1000, generala,
    generalruleUd, valueso, w → 2 π f, ZC → ZCexample1, K1 → 2 / 3, K2 → K1}];
```

Now finding optimal turn number:

```
testPb = Pa //. case6general // N;
```

```
Plot3D[testPb, {N, 10, 500}, {f, 40000, 200000}, PlotPoints → 100, PlotRange → All]
```




```
- SurfaceGraphics -
```

■ Rod diameter

```
case8polymer = {EP → 3 * 10^9,
   a21 → 0.8, a22 → 0.04,
   νP → 0.3,
   ρP → 1100 };
```

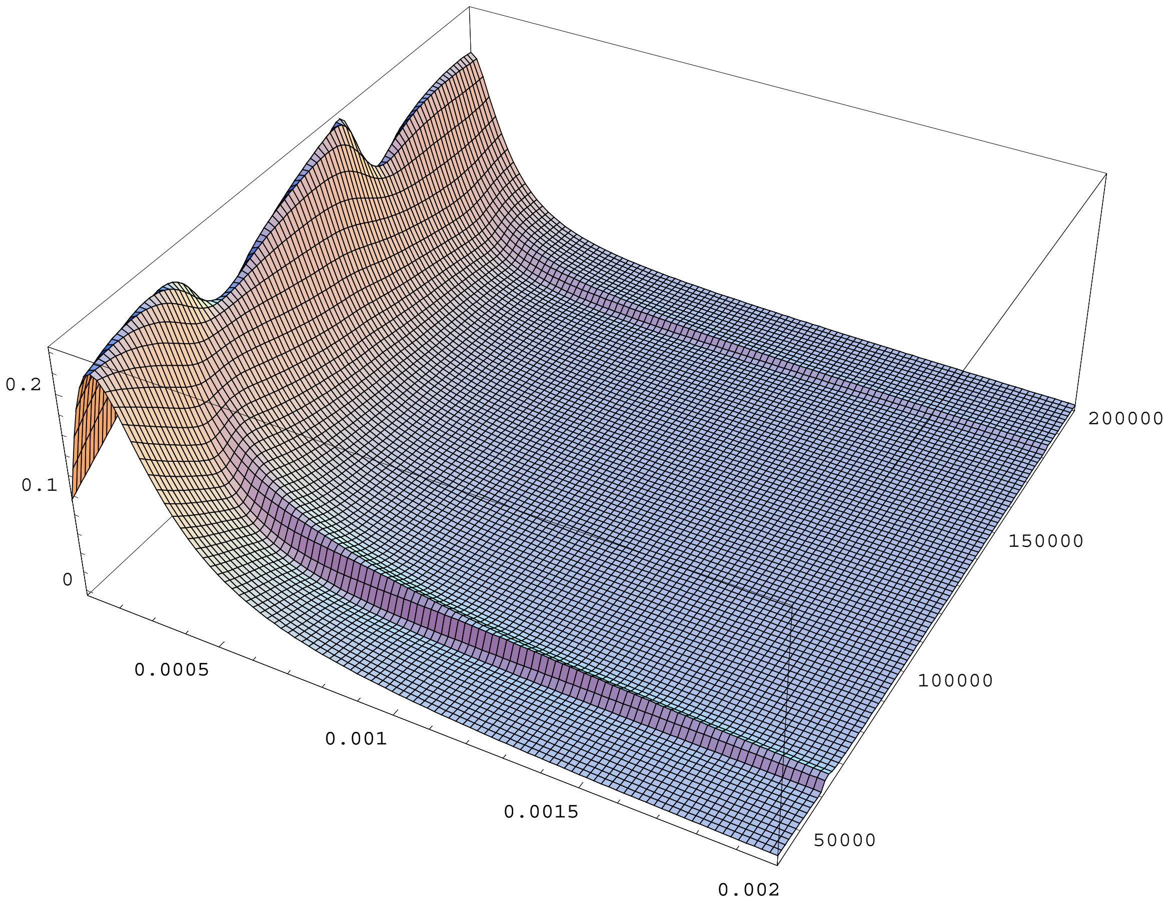
```
case8general = Flatten[{case8polymer, lRod → 40 / 1000, lCoil1 → 1 / 2 * lRod - 6 / 1000,
    lCoil2 → lCoil1, x1 → 3 / 1000, x2 -> 1 / 2 * lRod + x1, N → 500, N2 → 150,
    generala, generalruleUd, valueso, w → 2 π f, ZC → ZPx + 5, K1 → 2 / 3, K2 → K1}];
```

```
ZC //. case8general
```

$$5 - \frac{1.04937\times10^{9}\, i \sqrt{r^2}}{f} + 100000 \sqrt{330}\, \pi r^2 \left(0.902434 + 1.6982\times10^{-6} f^2 r^2\right)$$

```
testP2 = P2a //. case8general // N;
```

```
Plot3D[testP2, {r, 1 / 10 / 1000, 2 / 1000},
 {f, 40000, 200000}, PlotPoints → 100, PlotRange → All]
```



- SurfaceGraphics -