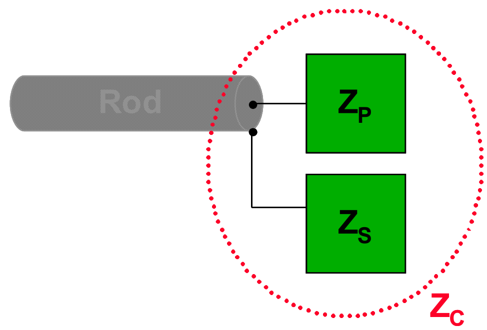
Contact Impedance of curing polymers

■ Definition of complex contact impedance at tip of rod:

Model simplifications:
It is assumed that the sealant as well as the polymer is in contact with the rod only at the end of the rod.Therefore an overall impedance ZC is defined as a second boundary condition:



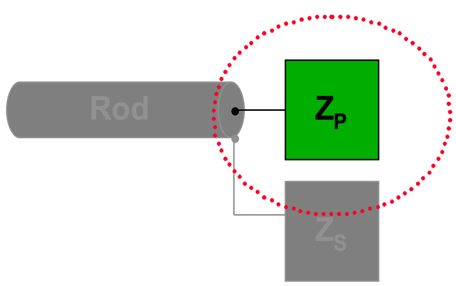

```
ZC = ZP + ZS;
```

With:
ZP: Impedance of polymer.
ZS: Impedance of sealant.

Assumption for simplification:

```
ZS = 0;
```

■ Definition of complex contact impedance of polymer:



Assumption according to Gladwell's derivation of contact impedance problems:

$$ZPx := rP + i\,w\,mP - \frac{i}{w\,qP}$$

With:

r_P: Resistive part (radiation) (rs referring to Gladwell)
mP: Mass (ms refering to Gladwell)
q_P: Surface compliance (qs referring to Gladwell)
The index P stands for "Polymer".

■ Resistive Part:

$$rP := \sqrt{\rho P * EP}\ Ac\ (A1 + B1 * \beta^2)$$

With:

ρ_P: Density of Polymer
A_c: Contact area.
Ai: Functions of Poisson-ratio. See below for details.

Actually the upper equation is a truncated series with addends with higher orders of β. The truncated approximation is only good for values of β which are less than 1. This is definitely not always the case during the whole curing process. (Later a minimum value for EP will be calculated for which this model can be used!)
β is actually a kind of wave number:

$$\beta = rc * w / cPs$$

$$\frac{rc\ w}{cPs}$$

This is the radius of the circular contact area:

$$rc = \sqrt{Ac / \pi}$$

$$\frac{\sqrt{A}}{\sqrt{\pi}}$$

Shear wave velocity:

$$cPs = \sqrt{\frac{GP}{\rho P}}$$

$$\sqrt{\frac{GP}{\rho P}}$$

The Shear modulus GP is related to the Young's modulus EP and the Poisson-ratio:

$$GP = \frac{EP}{2\ (1 + \nu P)}$$

$$\frac{EP}{2\ (1 + \nu P)}$$

The coefficients for rP:

$$A1 = a21 * \frac{2\sqrt{2}}{\pi\,(1-\nu P)\,\sqrt{1+\nu P}}$$

$$\frac{2\sqrt{2}\ a21}{\pi\,(1-\nu P)\,\sqrt{1+\nu P}}$$

$$B1 = a22 * \frac{2\sqrt{2}}{\pi\,(1-\nu P)\,\sqrt{1+\nu P}}$$

$$\frac{2\sqrt{2}\ a22}{\pi\,(1-\nu P)\,\sqrt{1+\nu P}}$$

With:

aij: Functions of Poisson-Ratio.

Note: A1 is about 1, and $A2 * \beta^2$ *is between 0.01 (low frequency of about 60kHz and high Young's-modulus) and 0.30 (frequencies up to 150 kHz and low Young's-modulus).*

■ Surface Compliance

$$qP := \frac{\sqrt{\pi}}{2}\ \frac{1-\nu P^2}{EP\,\sqrt{Ac}}$$

ZPx

$$\mathrm{i}\ mP\ w - \frac{2\,\mathrm{i}\,\sqrt{A}\ EP}{\sqrt{\pi}\ w\,(1-\nu P^2)} + A\sqrt{EP\,\rho P}\left(\frac{2\sqrt{2}\ a21}{\pi\,(1-\nu P)\,\sqrt{1+\nu P}} + \frac{4\sqrt{2}\ A\ a22\ w^2\,\sqrt{1+\nu P}\ \rho P}{EP\,\pi^2\,(1-\nu P)}\right)$$

■ Mass mP

According to Gladwell the influence of mP is, compared to qP, neglectable. This is not true at the end of the curing process, when this model is valid (see below). (At the beginning of the curing process it may play an important role, but this is not discussed here.)

```
mP = 0;
```

■ Verification:

■ Simulating a curing process

■ Epoxy Loctite Hysol

■ SC 15

```
ruleCureSC15 = {
  EPfinal → 2.7 * 10^9,
  νPfinal → 0.37,
  ρP → 1.2 * 1000,
  tcured -> 60 * 60 * 60}
```

{EPfinal → 2.7×10^9, νPfinal → 0.37, ρP → 1200., tcured → 216000}

■ Mechanical Impedance Plots

■ Electrical impedance Plots

■ Amplitude during curing process (general values)

■ ZP Example

■ Determining E-modulus of SC15

Using only the complex part of the contact impedance:

$$\text{ZPx1i} = \text{w mP} - \frac{1}{\text{w qP}} \text{ /. } \nu\text{P} \to \nu\text{Pfinal /. ruleCureSC15 /. w} \to 2\,\pi\,\text{f /. Ac} \to \text{A //. general}$$

$$-\frac{0.000368798\,\text{EP}}{\text{f}}$$

```
sol11 = Solve[{ZPtest == ZPx1i}, {EP}]
```

{{EP → -2711.51 f ZPtest}}

```
sol11 /. f → 65000 /. ZPtest → -10
```

{{EP → 1.76248×10^9}}