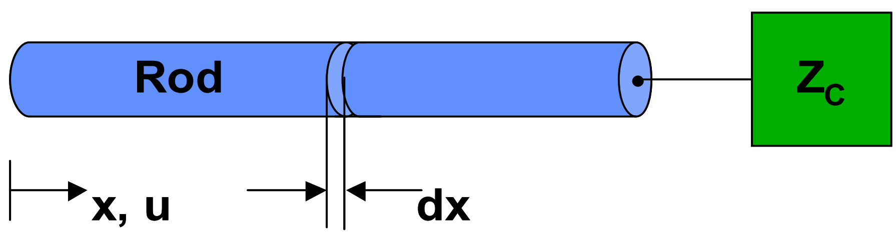
Model <1> for Magnetostrictive Rod

In this notebook the impedance of an ideal solenoid around a magnetostrictive rod is derived. Also derived is the amplitude of oscillations of the rod, dependant on the applied coil voltage. The model is only valid for steady-state oscillations, no electromechanical losses, and an ideal magnetic flux return path (that means the magnetic permeability μ of the surrounding medium has to be significantly higher than the permeability of the rod itself).

Additionally this model simulates the electric circuit for a solenoid connected to a function generator, a voltage-source with an internal resistance.

■ Initialization for Mathematica

■ General equations



■ Newton's second law

Newton's second law, reduced to a one-dimensional problem

In[19]:= `newton2 = ∂x σ[x, t] == ρR * ∂t,t u[x, t]`

Out[19]= $\sigma^{(1,0)}[x, t] == \rho R\, u^{(0,2)}[x, t]$

■ Strain ϵ

Strain is the first partial derivative of displacement, with respect to the position:

```
In[20]:= ϵ[x_, t_] = ∂x u[x, t]

Out[20]= u^(1,0)[x, t]
```

■ Stress σ

The following equation is designated as the **first magnetostrictive equation**. It is based on a linear model of magnetostriction. The classical stress-strain relationship is extended by a magnetostrictive relation, hence the following expression describes an association between mechanical and electromagnetic properties of the system::

```
In[21]:= σ[x_, t_] := ER ϵ[x, t] - θ B[t]
```

With:

σ: stress
ϵ: strain
ER: E-modulus of the magnetostrictive rod (under constant magnetic flux)
θ: "classical" magnetostrictive constant, unit $\frac{\text{Pascal}}{\text{Tesla}}$
B: Magnetic flux density (SI unit Tesla) in the rod. Please note that B is assumed to be constant over the length of the rod. See Chapter xxx for details.

Verification:

```
In[22]:= σ[x2, t2]

Out[22]= -θ B[t2] + ER u^(1,0)[x2, t2]
```

■ Differential equation for wave propagation

```
In[23]:= newton2

Out[23]= ER u^(2,0)[x, t] == ρR u^(0,2)[x, t]
```

■ Solution u for displacement in magnetostrictive rod

The displacement is assumed to be function of position multiplied by function of time:

```
In[24]:= u[x_, t_] := X[x] * T[t]
```

Function of time:

```
In[25]:= T[t_] := e^(i w t)
```

Verification:

```
In[26]:= Simplify[newton2]

Out[26]= e^(i t w) (w^2 ρR X[x] + ER X''[x]) == 0
```

The term with the magnetic flux density disappered because $\partial_x B$=0 and according to Maxwell div(B)=0

The function of position is assumed to be standing wave, the coefficients a1 and a2 can be complex (?????????)

```
In[27]:= X[x_] := a1 * Cos[K x] + a2 * Sin[K x]
```

The wave number K:

```
In[28]:= K = w / cR
```

Out[28]= $\frac{w}{cR}$

With:

K: Wave number.
w: Circular frequency of oscillation.
cR: Sound velocity in magnetostrictive rod.

Verification:

Newton's second law (equation "newton2") will now be verified with the solutions defined for u:

In[29]:= **Simplify[newton2 /. cR -> $\sqrt{\text{ER} / \rho\text{R}}$]**

Out[29]= True

■ Velocity v

Velocity is the first derivative of the displacement, with respect to time:

In[30]:= **v[x_, t_] = ∂_t u[x, t]**

Out[30]= $i\, e^{i t w}\, w \left(a1 \operatorname{Cos}\left[\frac{w x}{cR}\right] + a2 \operatorname{Sin}\left[\frac{w x}{cR}\right]\right)$

■ Electromagnetic relations (part 1)

■ Relation between Magnetic Flux Density B and properties of the solenoid

Based on **Faraday's law** the following equation is valid for an **ideal solenoid**:

In[31]:= **B[t_] := $\frac{1}{i\, w\, N\, A}$ * U[t]**

With:

B: Magnetic flux density in the rod.

N: Number of turns of the solenoid

A: Cross-sectional area of the solenoid.

Please note that the magnetic flux density is related to the voltage, not to the current going through the solenoid!

Actually a constant magnetic field for biassing the magnetostrictive rod is excluded from this model. Therefore B and H mean just the alternating components.

■ Voltage drop function

The voltage drop at the solenoid has to be a sine-shaped function when the system is in steady state. Therefore the time function T[t] (former used for the displacement) is also used for the voltage. However, there may be a phase shift between the displacement and the voltage.

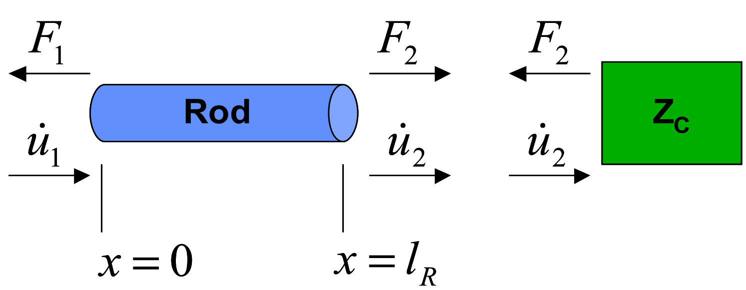
```
In[32]:= U[t_] := Ud * T[t]
```

With:

Ud: Complex amplitude of the voltage function.

T: $T = e^{i w t}$

■ Boundary conditions for amplitude coefficients *a1* and *a2*



■ First boundary condition (determining coefficient a2)

No stress at free end of rod (x=0) at all times:

```
In[33]:= BC1 =  σ[0, t] == 0
```

Out[33]= $\frac{a2\, e^{i t w}\, ER\, w}{cR} + \frac{i\, e^{i t w}\, Ud\, \theta}{A\, N\, w} == 0$

Solve for coefficient a2 and create a rule:

```
In[34]:= rulea2 = Flatten[Solve[BC1, a2]]
```

Out[34]= $\left\{a2 \to -\frac{i\, cR\, Ud\, \theta}{A\, ER\, N\, w^2}\right\}$

a2 is the amplitude of the sine function of the displacement,SI unit meter.

■ Second boundary condition (determining coefficient a1)

It is assumed that the sealant as well as the polymer is in contact with the rod only at the end of the rod **(keen simplification!)**. Therefore an overall impedance ZC is defined as a second boundary condition:

In[35]:= $\text{BC2} = -\text{ZC} == \text{Ac} * \frac{\sigma[\text{lRod}, t]}{v[\text{lRod}, t]};$

With:
ZC: Complex mechanical contact impedance of polymer and sealant, SI unit $\frac{N\,s}{m} = \frac{kg}{s}$
Ac: Contact area.

Model simplifications:

```
In[36]:= Ac = A;
```

Equals cross-sectional area of rod and solenoid in this model.

Derivation of a1:

```
In[37]:= BC2 = Simplify[BC2];
```

Solve for coefficient a1 and create a rule:

```
In[38]:= rulea1 = Simplify[Flatten[Solve[BC2, a1]]]
```

Out[38]= $\left\{a1 \to -\frac{cR\, Ud\, \theta - i\, A\, a2\, ER\, N\, w^2\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] + a2\, cR\, N\, w^2\, ZC\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]}{N\, w^2\, \left(cR\, ZC\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] + i\, A\, ER\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]\right)}\right\}$

Apply rule for coefficient a2 on coefficient a1:

```
In[39]:= rulea1 = rulea1 /. rulea2
```

Out[39]= $\left\{a1 \to -\frac{cR\, Ud\, \theta - cR\, Ud\, \theta\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] - \frac{i\, cR^2\, Ud\, ZC\, \theta\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]}{A\, ER}}{N\, w^2\, \left(cR\, ZC\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] + i\, A\, ER\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]\right)}\right\}$

```
In[40]:= rulea1 = Simplify[rulea1]
```

Out[40]= $\left\{a1 \to \frac{cR\, Ud\, \theta\, \left(-A\, ER + A\, ER\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] + i\, cR\, ZC\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]\right)}{A\, ER\, N\, w^2\, \left(cR\, ZC\, \text{Cos}\left[\frac{lRod\, w}{cR}\right] + i\, A\, ER\, \text{Sin}\left[\frac{lRod\, w}{cR}\right]\right)}\right\}$

a1 is the amplitude of the cosine function of the displacement,SI unit meter.

a1 in case of ZC=0:

In[41]:= `rulea1ZC0 = Simplify[rulea1 /. ZC → 0]`

Out[41]= $\left\{\text{a1} \to \frac{i\ \text{cR Ud}\ \theta\ \text{Tan}\left[\frac{\text{lRod w}}{2\ \text{cR}}\right]}{\text{A ER N w}^2}\right\}$

Verification for the case that ZC=0 *and w is first resonant frequency of rod:*

In[42]:= $\text{rulea1ZC0 /. w} \to 2\pi * \left(\frac{\text{cR}}{2\ \text{lRod}}\right)$

Out[42]= {a1 → ComplexInfinity}

Verification for the case that ZC=0 *and w is second resonant frequency of rod:*

In[43]:= $\text{rulea1ZC0 /. w} \to 2\pi * \left(\frac{2\ \text{cR}}{2\ \text{lRod}}\right)$

Out[43]= {a1 → 0}

Verification for limit values of ZC:

In[44]:= `Simplify[Limit[a1 /. rulea1, ZC → ∞]]`

Out[44]= $\frac{i\ \text{cR Ud}\ \theta\ \text{Tan}\left[\frac{\text{lRod w}}{\text{cR}}\right]}{\text{A ER N w}^2}$

In[45]:= `Simplify[Limit[a1 /. rulea1, ZC → i * ∞]]`

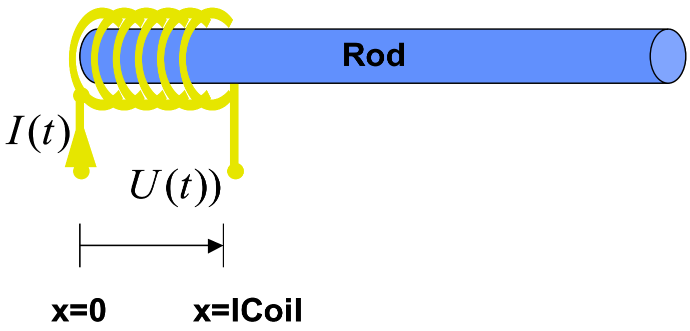
Out[45]= $\frac{i\ \text{cR Ud}\ \theta\ \text{Tan}\left[\frac{\text{lRod w}}{\text{cR}}\right]}{\text{A ER N w}^2}$

■ Absolute amplitude of oscillations:

This is the absolute amplitude of the oscillations:

In[46]:= $\text{amp[a1\_, a2\_] :=} \sqrt{\text{Abs[a1]\^{}2 + Abs[a2]\^{}2}}$

■ Further electromagnetic relations (part 2)



■ Magnetic field strength H

The following equation is designated as the **second magnetostrictive equation** of the linear model. The classical relationship between magnetic field strength and flux density (B=μ H) is extended by strain

In[47]:= $H[x\_, t\_] := \frac{1}{\mu R} * B[t] - \theta \epsilon[x, t]$

With:
H: Magnetic field strength (SI unit $\frac{Ampere}{Meter}$). *The magnetic field strength*

Verification:

In[48]:= H[x, t]

Out[48]= $-\frac{i\, e^{i t w}\, Ud}{A N w \mu R} - e^{i t w} \theta \left(\frac{a2\, w \operatorname{Cos}[\frac{w x}{cR}]}{cR} - \frac{a1\, w \operatorname{Sin}[\frac{w x}{cR}]}{cR} \right)$

Contrary to the flux density, the field strength varies in between the rod. Therefore H has to be integrated over the length of the coil. In the following consideration the solenoid is placed at the free end of the rod, with a yet unknown length lCoil1:

In[49]:= $Hq[t\_] := \int_{x1}^{x1+lCoil1} \frac{H[x, t]}{lCoil1}\, dx$

With:
Hq: Average magnetic flux density in the solenoid
lCoil1: Length of the solenoid
x1: Location of the coil

Verification:

In[50]:= **Hq[t]**

$$\text{Out[50]}= \frac{1}{\text{lCoil1}}\left(-\frac{i\, e^{i\,t\,w}\,\text{lCoil1 Ud}}{\text{A N w}\,\mu\text{R}} + \text{a1}\, e^{i\,t\,w}\,\theta\,\text{Cos}\left[\frac{\text{w x1}}{\text{cR}}\right] - \text{a1}\, e^{i\,t\,w}\,\theta\,\text{Cos}\left[\frac{\text{w (lCoil1 + x1)}}{\text{cR}}\right] + \text{a2}\, e^{i\,t\,w}\,\theta\,\text{Sin}\left[\frac{\text{w x1}}{\text{cR}}\right] - \text{a2}\, e^{i\,t\,w}\,\theta\,\text{Sin}\left[\frac{\text{w (lCoil1 + x1)}}{\text{cR}}\right]\right)$$

■ Current i

The current i is proportional to the (average) magnetic field strength, assuming an ideal solenoid:

$$\text{In[51]:= } \mathbf{i[t\_] := \frac{Hq[t] * lCoil1}{N} \Big/ K1}$$

With:

i: Current through solenoid

K1: A combined factor of geometry factor and field factor of the coil. For K=1 it becomes an equation for an ideal solenoid.

Verification:

In[52]:= **i[t]**

$$\text{Out[52]}= \frac{1}{\text{K1 N}}\left(-\frac{i\, e^{i\,t\,w}\,\text{lCoil1 Ud}}{\text{A N w}\,\mu\text{R}} + \text{a1}\, e^{i\,t\,w}\,\theta\,\text{Cos}\left[\frac{\text{w x1}}{\text{cR}}\right] - \text{a1}\, e^{i\,t\,w}\,\theta\,\text{Cos}\left[\frac{\text{w (lCoil1 + x1)}}{\text{cR}}\right] + \text{a2}\, e^{i\,t\,w}\,\theta\,\text{Sin}\left[\frac{\text{w x1}}{\text{cR}}\right] - \text{a2}\, e^{i\,t\,w}\,\theta\,\text{Sin}\left[\frac{\text{w (lCoil1 + x1)}}{\text{cR}}\right]\right)$$

This shows that the current is approximately proportional to the amplitude coefficient a1 (a2 is small near the resonance frequencies)!

Including the displacement coefficients which where derived by using the boundary conditions:

In[53]:= **ia[t_] = FullSimplify[i[t] /. Flatten[{rulea1, rulea2}]]**

$$\text{Out[53]}= -\Bigg(e^{i\,t\,w}\,\text{Ud}\Bigg(\text{cR ER lCoil1 w ZC Cos}\left[\frac{\text{lRod w}}{\text{cR}}\right] + i\Bigg(\text{A cR ER}\,\theta^2\,\mu\text{R Cos}\left[\frac{\text{w (lRod - x1)}}{\text{cR}}\right] - \text{A cR ER}\,\theta^2\,\mu\text{R Cos}\left[\frac{\text{w x1}}{\text{cR}}\right] + \text{A cR ER}\,\theta^2\,\mu\text{R Cos}\left[\frac{\text{w (lCoil1 + x1)}}{\text{cR}}\right] - \text{A cR ER}\,\theta^2\,\mu\text{R Cos}\left[\frac{\text{w (lCoil1 - lRod + x1)}}{\text{cR}}\right] + \text{A ER}^2\,\text{lCoil1 w Sin}\left[\frac{\text{lRod w}}{\text{cR}}\right] + i\,\text{cR}^2\,\text{ZC}\,\theta^2\,\mu\text{R Sin}\left[\frac{\text{w (lRod - x1)}}{\text{cR}}\right] + i\,\text{cR}^2\,\text{ZC}\,\theta^2\,\mu\text{R Sin}\left[\frac{\text{w (lCoil1 - lRod + x1)}}{\text{cR}}\right]\Bigg)\Bigg)\Bigg) \Big/ \left(\text{A ER K1 N}^2\,\text{w}^2\,\mu\text{R}\left(-i\,\text{cR ZC Cos}\left[\frac{\text{lRod w}}{\text{cR}}\right] + \text{A ER Sin}\left[\frac{\text{lRod w}}{\text{cR}}\right]\right)\right)$$

With:

ia: Current through solenoid

Verification:

```
In[54]:= (*FullSimplify[ia[t]/.{ZC→0,lCoil1->lRod}]*)
```

```
In[55]:= (*FullSimplify[ia[t]/.{ZC→0}]*)
```

■ Electric Impedance Z

```
In[56]:= Z := Simplify[U[t] / ia[t]]
```

With:

Z: Electric impedance of coil

Verification:

```
In[57]:= Simplify[Z /. {ZC → 0, lCoil1 -> lRod}]
```

Out[57]=
$$\frac{i\, A\, ER\, K1\, N^2\, w^2\, \mu R\, \mathrm{Sin}\left[\frac{lRod\, w}{cR}\right]}{-4\, cR\, \theta^2\, \mu R\, \mathrm{Cos}\left[\frac{w\, x1}{cR}\right]\, \mathrm{Sin}\left[\frac{lRod\, w}{2\, cR}\right]^2 + ER\, lRod\, w\, \mathrm{Sin}\left[\frac{lRod\, w}{cR}\right]}$$

■ Magnetostrictive power

Electric power which goes into the solenoid (complex - real part is active power, complex part is reactive power):

```
In[58]:= P[t_] = U[t] * ia[t];
```

This is the **effective** (time independent) apparent power:

```
In[59]:= Pa = 1 / 2 * Simplify[Abs[U[t]]] * Simplify[Abs[ia[t]], cond1];
```

■ Model verification

■ Unit Verification

■ Numeric calculations

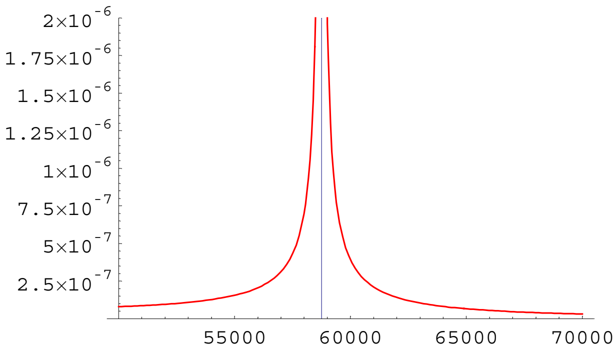
■ Plotting

■ Amplitude at Constant Voltage (Artificial Case!)

```
In[79]:= test1i = Abs[a1] /. valuerulea /. {w → 2 π f, ZC → 0, Ud -> 10} // N;
```

```
In[80]:= test2i = Abs[a1] /. valuerulea /. {w → 2 π f, ZC → ZCexample1, Ud -> 10} // N;
```

```
In[81]:= Plot[{test1i, test2i}, {f, 50000, 70000},
           PlotPoints → 150, PlotRange → {0, 2 / 1000000}, PlotStyle →
            {{Thickness[.004], RGBColor[1, 0, 0]}, {Thickness[.002], RGBColor[0, 1, 0]}},
           GridLines → {{58750, 117.5}, None}, Axes → {True, True}]

         Plot::plnr : test2i is not a machine-size real number at f = 50000.00013422819`.

         Plot::plnr : test2i is not a machine-size real number at f = 50130.685610436245`.

         Plot::plnr : test2i is not a machine-size real number at f = 50273.20955659396`.

         General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```




```
Out[81]= - Graphics -
```

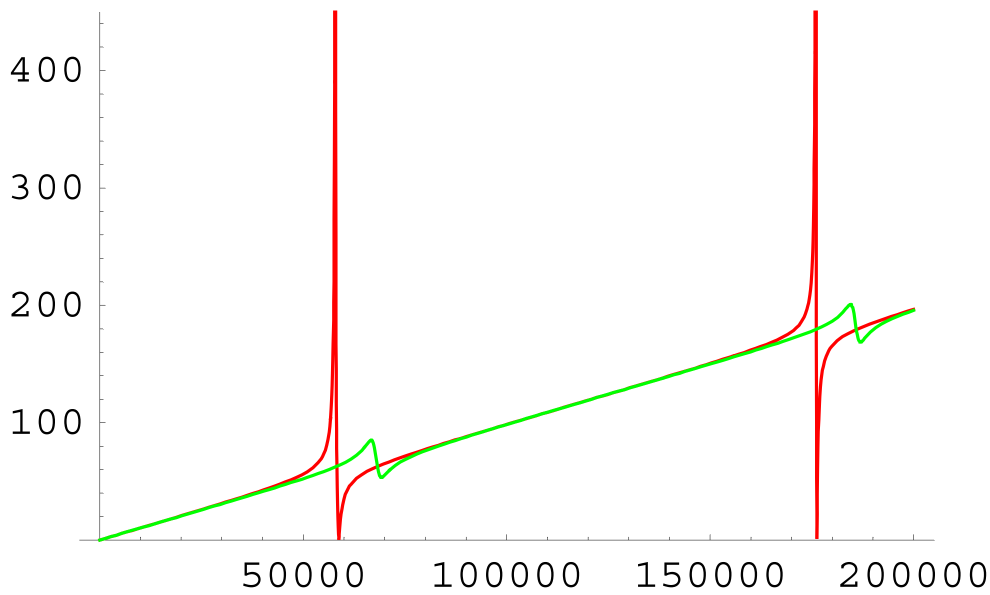
■ Impedance Plots

■ Plotting the electrical Impedance, once with long coil and once with short coil

■ Plotting the electrical Impedance, once without and once with load ZC:

```
In[85]:= test3 = Abs[Z /. lCoil1 → lRod /. values /. {ZC → 0, w → 2 π f}] // N;

In[86]:= test4 = Abs[Z /. lCoil1 → lRod /. values /. {ZC → 50 - ⅈ 300, w → 2 π f}] // N;
```

```
In[87]:= Plot[{test3, test4}, {f, 0, 200000}, PlotPoints → 150, PlotRange → {0, 450},
          AxesLabel → {"", ""}, TextStyle → {FontSize → 20}, PlotStyle →
           {{Thickness[.004], RGBColor[1, 0, 0]}, {Thickness[.004], RGBColor[0, 1, 0]}}]
```




```
Out[87]= - Graphics -
```

■ Further electric relations (part 3)

■ Amplitude at HP Function Generator

The Voltage drop at the solenoid depends on the internal resistence and on the current:

```
In[88]:= Kirchhoff1 = U0[t] == i[t] (Ri + Z);
```

The internal voltage-source of the generator and the voltage at the solenoid have the same phase angle (because of the true ohmic internal resistance ri) ans the sam timefunction:

```
In[89]:= U0[t_] = U0d * T[t]
```

Out[89]= $e^{i t w}$ U0d

Solve for the voltage amplitude at the solenoid:

```
In[90]:= Kirchhoff1 = Simplify[Kirchhoff1 /. Flatten[{rulea1, rulea2}]];
```

```
In[91]:= ruleUd = Flatten[Simplify[Solve[Kirchhoff1, Ud]]]
```

Out[91]= $\left\{Ud \to \left(A\, ER\, K1\, N^2\, U0d\, w^2\, \mu R \left(-i\, cR\, ZC\, Cos\left[\frac{lRod\, w}{cR}\right] + A\, ER\, Sin\left[\frac{lRod\, w}{cR}\right]\right)\right) \Big/ \left(-cR\, ER\, w\, ZC\, (lCoil1\, Ri + i\, A\, K1\, N^2\, w\, \mu R)\, Cos\left[\frac{lRod\, w}{cR}\right] - i\, A\, cR\, ER\, Ri\, \theta^2\, \mu R\, Cos\left[\frac{w\,(lRod - x1)}{cR}\right] + i\, A\, cR\, ER\, Ri\, \theta^2\, \mu R\, Cos\left[\frac{w\, x1}{cR}\right] - i\, A\, cR\, ER\, Ri\, \theta^2\, \mu R\, Cos\left[\frac{w\,(lCoil1 + x1)}{cR}\right] + i\, A\, cR\, ER\, Ri\, \theta^2\, \mu R\, Cos\left[\frac{w\,(lCoil1 - lRod + x1)}{cR}\right] - i\, A\, ER^2\, lCoil1\, Ri\, w\, Sin\left[\frac{lRod\, w}{cR}\right] + A^2\, ER^2\, K1\, N^2\, w^2\, \mu R\, Sin\left[\frac{lRod\, w}{cR}\right] + cR^2\, Ri\, ZC\, \theta^2\, \mu R\, Sin\left[\frac{w\,(lRod - x1)}{cR}\right] + cR^2\, Ri\, ZC\, \theta^2\, \mu R\, Sin\left[\frac{w\,(lCoil1 - lRod + x1)}{cR}\right]\right)\right\}$

Ud in case of constant lCoil:

```
In[92]:= ruleUdlc = ruleUd /. values /. {lCoil1 → lRod} /. values;
```

```
In[93]:= ruleUdlc2 = ruleUd /. values /. {lCoil1 → lRod / 2} /. values;
```

The following rule sets a1 and a2 for the HP function generator. Open is: lCoil1, Zp, w

```
In[94]:= valueruleaHP = valuerulea /. ruleUd /. values;
```

In case of lcoil=lRod:

```
In[95]:= valueruleaHPlc = Simplify[valuerulea /. ruleUd /. lCoil1 → lRod /. values];
```

In case of lcoil=lRod / 3:

```
In[96]:= valueruleaHPlc3 = Simplify[valuerulea /. ruleUd /. lCoil1 → lRod / 3 /. values];
```

■ Plotting (part 2)

■ Amplitude and Phase

```
In[97]:= test3 = Abs[a1 /. rulea1 /. values /. ZC → 0 /. w → 2 π f] // N
```

Out[97]= $\frac{1}{f^2}\left(1.88477\times10^{-6}\, Abs[Ud\,(-2.51327\times10^{6} + 2.51327\times10^{6}\, Cos[0.0000534739\, f])\, Csc[0.0000534739\, f]]\right)$

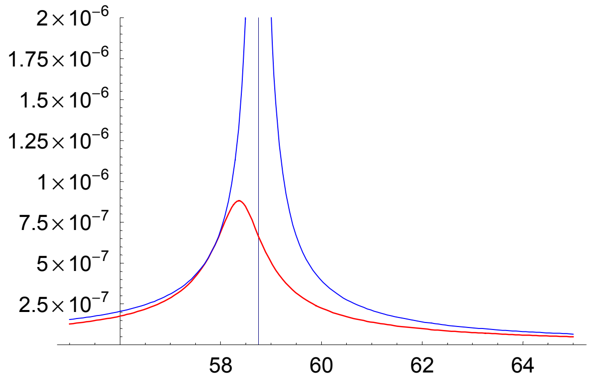
```
In[98]:= test1 = Abs[a1 /. valueruleaHPlc /. ZC → 0 /. w → 2 π f] // N;
```

```
In[99]:= test1b = Abs[a1 /. valueruleaHPlc /. ZC → ZCexample1 /. w → 2 π f] // N;
```

```
General::spell :
 Possible spelling error: new symbol name "test1b" is similar to existing symbols {test1, test1i}.
```

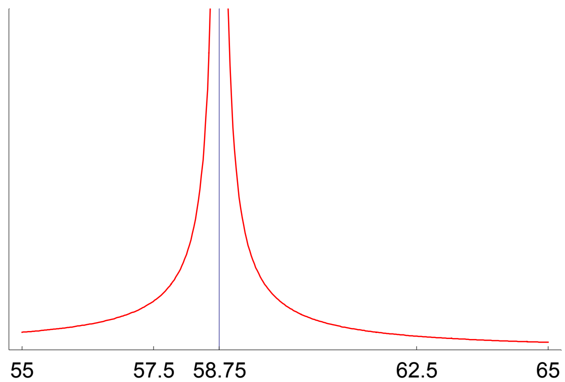
```
In[100]:= test2 = Arg[a1 /. valueruleaHPlc /. ZC → 0 /. w → 2 π f] // N;
```

```
In[101]:= pl1a = Plot[{test1 /. f → fk * 1000, test1b /. f → fk * 1000,
             test1i /. f → fk * 1000, test2i /. f → fk * 1000}, {fk, 55, 65}, PlotPoints → 150,
            PlotRange → {0, 0.000002}, PlotStyle → {{Thickness[.003], RGBColor[1, 0, 0]},
              {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0, 1]},
              {Thickness[.002], RGBColor[1, 0, 1]}, {Thickness[.002], RGBColor[0.2, 0.8, 0]},
              {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0.6, 0.4]}},
            TextStyle → {FontFamily → "Helvetica", FontSize → 10},
            GridLines → {{58.75, 117.5}, None}, Axes → {True, True}]

      Plot::plnr : test1b /. f → fk 1000 is not a machine-size real number at fk = 55.000000067114094`.

      Plot::plnr : test1b /. f → fk 1000 is not a machine-size real number at fk = 55.065342805218116`.

      Plot::plnr : test1b /. f → fk 1000 is not a machine-size real number at fk = 55.13660477829698`.

      General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```

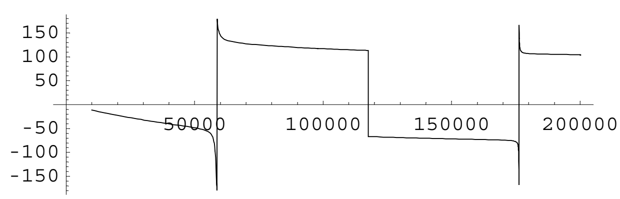



```
Out[101]= - Graphics -
```

```
In[102]:= pl1b = Plot[{test1i /. f → fk * 1000, test2i /. f → fk * 1000},
     {fk, 55, 65}, PlotPoints → 350, PlotRange → {0, 0.000003},
     PlotStyle → {{Thickness[.003], RGBColor[1, 0, 0]},
       {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0, 1]},
       {Thickness[.002], RGBColor[1, 0, 1]}, {Thickness[.002], RGBColor[0.2, 0.8, 0]},
       {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0.6, 0.4]}},
     TextStyle → {FontFamily → "Helvetica", FontSize → 10},
     GridLines → {{58.75, 117.5}, None}, Axes → {True, True},
     Ticks → {{55, 57.5, 58.75, 62.5, 65}, None}, AxesOrigin → {54.75, 0}]

Plot::plnr : test2i /. f → fk 1000 is not a machine-size real number at fk = 55.0000000286533`.

Plot::plnr : test2i /. f → fk 1000 is not a machine-size real number at fk = 55.027897071568766`.

Plot::plnr : test2i /. f → fk 1000 is not a machine-size real number at fk = 55.05832123772564`.

General::stop : Further output of Plot::plnr will be suppressed during this calculation.
```




```
Out[102]= - Graphics -

In[103]:= pl2 = Plot[{test2 / (2 π) * 360}, {f, 10000, 200000},
     PlotPoints → 150, PlotRange → Automatic, AspectRatio → 1 / 3]
```




```
Out[103]= - Graphics -
```

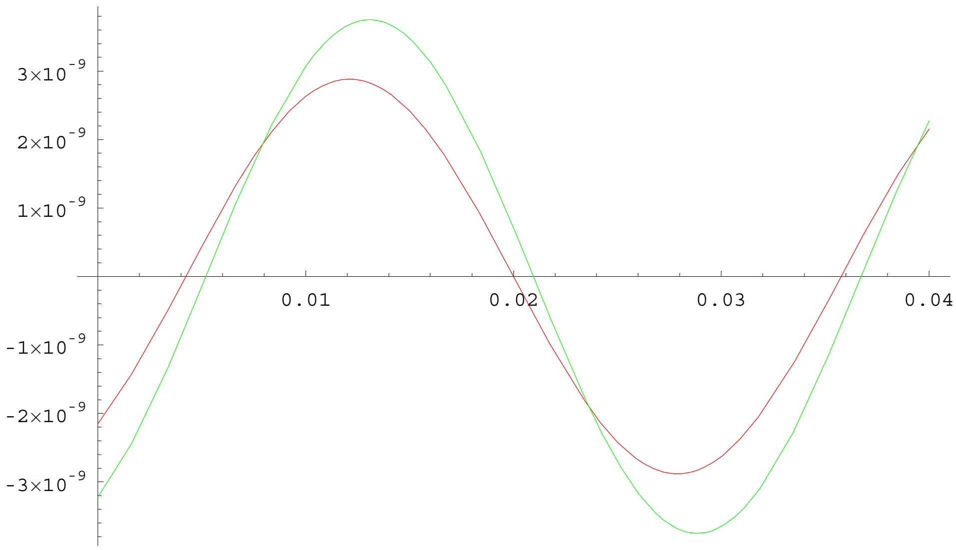
■ Amplitude over Length-coordinate (Function X[x])

Plotting the imaginary part of the function X at a certain frequency (once without ZC, once with ZC≠0),over its coordinate x:

```
In[104]:= test1 =
     Im[X[x] /. Flatten[{rulea1, rulea2}] /. ruleUdlc /. values /. ZC → 0        /. w → 2 π f /.
       f → 149000] // N;
```

In[105]:=
```
test2 =
  Im[X[x] /. Flatten[{rulea1, rulea2}] /. ruleUdlc /. values /. ZC → 0 + 250 ⅈ /. w → 2 π f /.
     f → 149000] // N;
```

In[106]:=
```
Plot[{test1, test2}, {x, 0, lRod /. values},
 PlotStyle →
  {{Thickness[.001], RGBColor[1, 0, 0]}, {Thickness[.001], RGBColor[0, 1, 0]}}]
```

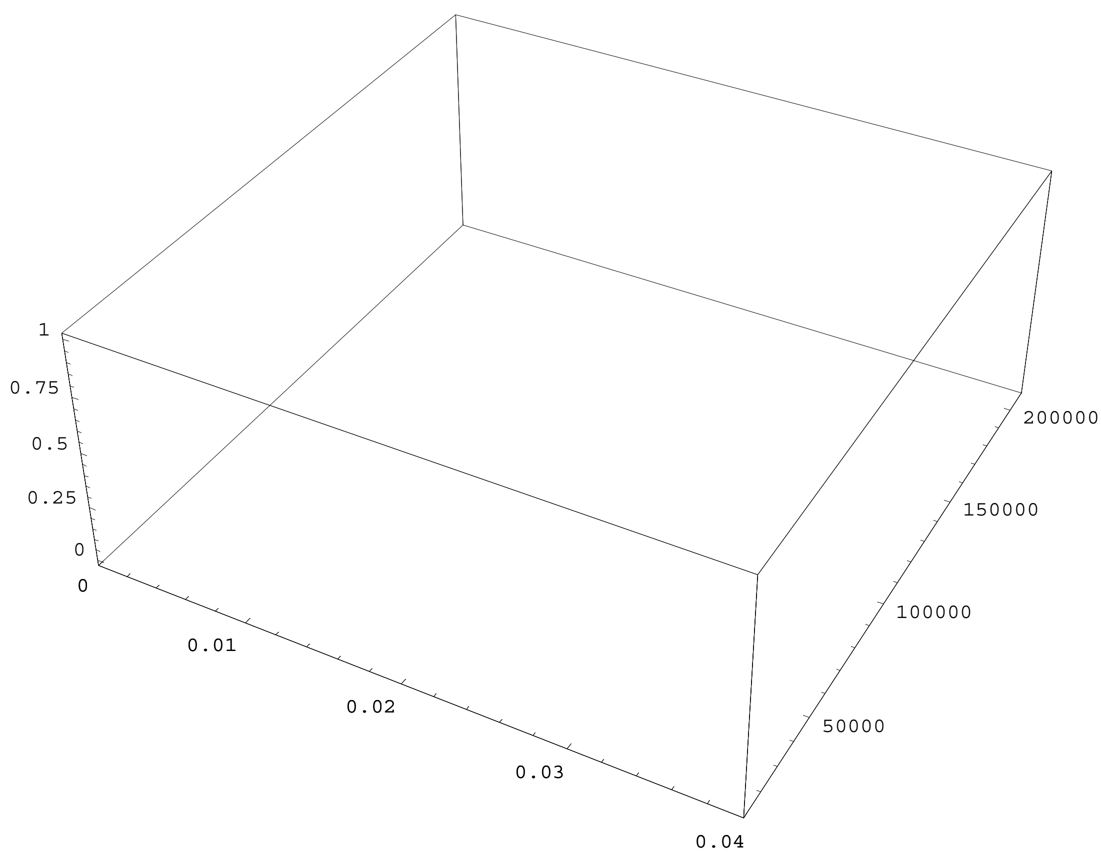


Out[106]= - Graphics -

Now plotting X over frequency:

In[107]:=
```
test4 = Im[Xv /. w → 2 π f];
```

```
In[108]:= Plot3D[test4, {x, 0, lRod /. values}, {f, 10000, 210000}, PlotPoints → 50]

Plot3D::plnc : test4 is neither a machine-size real number
   at {x, f}={0., 10000.} nor a list of a real number and a valid color directive.

Plot3D::plnc : test4 is neither a machine-size real number at
   {x, f}={0.000816327, 10000.} nor a list of a real number and a valid color directive.

Plot3D::plnc : test4 is neither a machine-size real number at
   {x, f}={0.00163265, 10000.} nor a list of a real number and a valid color directive.

General::stop : Further output of Plot3D::plnc will be suppressed during this calculation.

Plot3D::gval : Function value Im[Xv] at grid point xi = 1, yi = 1 is not a real number.

Plot3D::gval : Function value Im[Xv] at grid point xi = 1, yi = 2 is not a real number.

Plot3D::gval : Function value Im[Xv] at grid point xi = 1, yi = 3 is not a real number.

General::stop : Further output of Plot3D::gval will be suppressed during this calculation.
```




```
Out[108]= - SurfaceGraphics -
```

■ Amplitude components (sin and cos)

ZC<>0

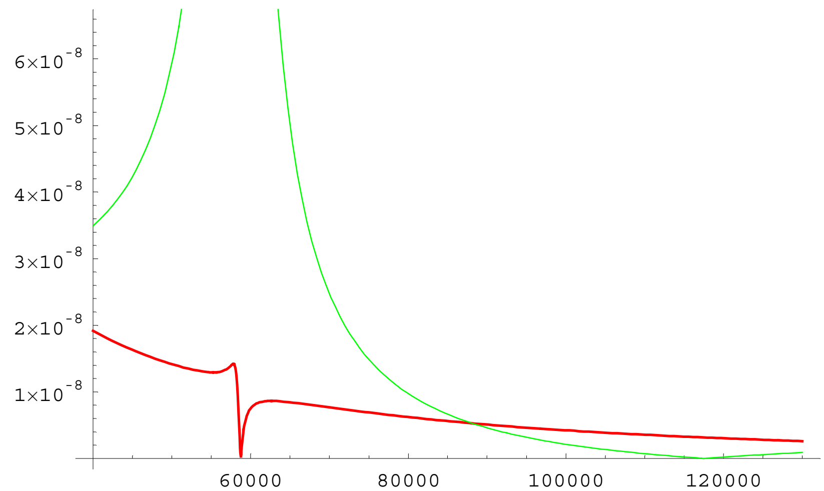
```
In[109]:= vas = a1 /. valueruleaHPlc
```

Out[109]= $\left(5875\, i\, \pi \left(-8000\,\pi + 8000\,\pi \,\text{Cos}\left[\frac{w}{117500}\right] + 47\, i\, ZC\, \text{Sin}\left[\frac{w}{117500}\right]\right)\right) \Big/ \left(-47\, i\, (-3125000\, i\, w\, ZC + \pi^2\, (1500000000000 + w^2\, ZC))\, \text{Cos}\left[\frac{w}{117500}\right] + 2000\,\pi\left(35250000000\, i\, \pi + (-12500000\, i\, w + 4\,\pi^2\, w^2 + 103546875\, ZC)\, \text{Sin}\left[\frac{w}{117500}\right]\right)\right)$

```
In[110]:= vas2 = a2 /. valueruleaHPlc
```

Out[110]= $\left(5875\,\pi \left(47\, ZC\, \text{Cos}\left[\frac{w}{117500}\right] + 8000\, i\, \pi\, \text{Sin}\left[\frac{w}{117500}\right]\right)\right) \Big/ \left(-47\, i\, (-3125000\, i\, w\, ZC + \pi^2\, (1500000000000 + w^2\, ZC))\, \text{Cos}\left[\frac{w}{117500}\right] + 2000\,\pi\left(35250000000\, i\, \pi + (-12500000\, i\, w + 4\,\pi^2\, w^2 + 103546875\, ZC)\, \text{Sin}\left[\frac{w}{117500}\right]\right)\right)$

```
In[135]:= Plot[{Abs[vas2 /. ZC → 0 /. w → 2 π f], Abs[vas /. ZC → 0 /. w → 2 π f]},
  {f, 40000, 130000}, PlotPoints → 150, PlotRange → Automatic, PlotStyle →
   {{Thickness[.004], RGBColor[1, 0, 0]}, {Thickness[.002], RGBColor[0, 1, 0]}}]
```



Out[135]= - Graphics -

Amplitude does not go to infinity, even under resonance conditions! This is related with the internal losses of the HP power supply. Setting the internal resistance Ri to zero would cause a constant voltage condition, with infinite current. See below and see above.

■ Absolute Amplitude Plots

■ Low mechanical impedance

Zero impedance (red):

```
In[112]:= vat1 = amp[a1, a2] /. valueruleaHPlc /. w → 2 π f * 1000 /. ZC → 0 // N;
```

Resistive impedance (green):

```
In[113]:= vat2 = amp[a1, a2] /. valueruleaHPlc /. w → 2 π f * 1000 /. ZC → 15 // N;

        General::spell : Possible spelling error: new symbol name "vat2" is similar to existing symbols {val2, vas2}.
```

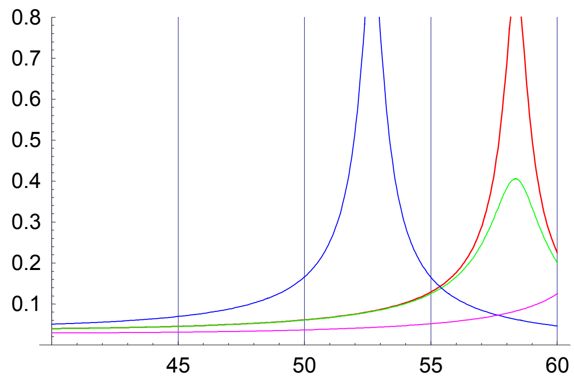
Resistive impedance + inertia term (blue):

```
In[114]:= vat3 = amp[a1, a2] /. valueruleaHPlc /. ZC → 0 + ⅈ w * 5 * 10^-4 /. w → 2 π f * 1000 // N;
          (*    ZC=15+ⅈ 30    *)
```

Resistive impedance + capacitive term (pink):

```
In[115]:= vat4 = amp[a1, a2] /. valueruleaHPlc /. ZC → - ⅈ/(2 * 10^-8 w) /. w → 2 π f * 1000 // N;
          (*    ZC=15-ⅈ 30    *)
```

```
In[116]:= pl1 = Plot[{10^6 vat1, 10^6 vat2, 10^6 vat3, 10^6 vat4}, {f, 40, 60}, PlotPoints → 350,
            PlotRange → {0, 0.8}, PlotStyle → {{Thickness[.003], RGBColor[1, 0, 0]},
              {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0, 1]},
              {Thickness[.002], RGBColor[1, 0, 1]}, {Thickness[.002], RGBColor[0.2, 0.8, 0]},
              {Thickness[.002], RGBColor[0, 1, 0]}, {Thickness[.002], RGBColor[0, 0.6, 0.4]}},
            TextStyle → {FontFamily → "Helvetica", FontSize → 10}, GridLines → {Automatic, None}]
```




```
Out[116]= - Graphics -
```

■ High mechanical impedance (limit values)

■ Receiving Coil

Note that the effect of a receiving coil is not mentioned in the equations above. Therefore they are only valid for a low power consumption at the receiving coil.

Index 2 for the receiving coil.

Faraday again:

```
In[123]:= U2[t] := ⅈ w N2 A B[t]
```

This shows that the voltage of the receiving coil is related to the voltage of exciting coil. This is the same equation like for a transformer. Therefore the magnetostrictive effect can only be measured because of the internal resistance of the voltage source.

In[124]:= U2[t]

Out[124]= $\frac{e^{i t w} N2\, Ud}{N}$

An other method to detect the magnetostrictive effect is by measuring the current at the receiving coil:

In[125]:= $i2[t\_] := \frac{Hq2[t] * lCoil2}{N2} * K2$

With the average field strength:

In[126]:= $Hq2[t\_] := \int_{x2}^{x2+lCoil2} \frac{H[x, t]}{lCoil2}\, dx$

```
In[127]:= i2a[t_] = Simplify[i2[t] /. rulea2 /. rulea1 /. ruleUd];
```

Now the power output of the receiving coil is calculated (complex - real part is active power, complex part is reactive power):

```
In[128]:= P2[t] = U2[t] * i2a[t];
```

This is the **effective** (time independent) apparent power:

```
In[129]:= P2a = 1 / 2 * Simplify[Abs[U2[t]], cond1] * Simplify[Abs[i2a[t]], cond1] /. ruleUd;
```

Further simplification, with dividing every complex variable into real and imaginary part (and into absolute part and argument, respectively):

```
In[130]:= P2as = ComplexExpand[P2a /. ZC → ZCr + i ZCi, TargetFunctions -> {Re, Im}];
```

Note that "ComplexExpand" assumes all vaiables to be real. This means we define the amplitude U0d to be real.

With:

ZC_R

ZC_i

Further simplification:

```
In[131]:= (*P2axs=Simplify[P2ax,cond1,Trig→False,TimeConstraint→50 ]*)
```

■ Zero Impedance -